DEPOSIT INSURANCE AGENCY
RUSSIAN FEDERATION

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Credit Risk Modeling

Literature Review

for the purposes of evaluation of DI Fund sufficiency on the basis of risk analysis

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TABLE OF CONTENTS

Preface

Background information

In 2006 Research and Guidance Committee of International Association of Deposit Insurers (IADI) initiated and its Executive Council approved establishment of a special subcommittee for developing a Research Paper on methodologies that can be used by deposit insurers to estimate their deposit insurance funds' sufficiency on the basis of risk analysis[1]. This paper , written to support the subcommittee's effort[2], represents a review of theoretical literature on credit risk analysis and modeling which can form a basis for practical implementation by deposit insurers This document will be considered as a complement to the Guidance prepared by the subcommittee.

We believe that this survey of theory and research on credit risk modeling will contribute to enhancement of practices of deposit insurers in the area of risk analysis and introducing/improving their methodology of risk analysis and its implications for managing and funding their deposit insurance funds.

Problem outline

A deposit insurance system (DIS) holds a portfolio of liabilities to the insured depositors contingent on the events of default. These liabilities are offset by the deposit insurance fund (DIF) which is funded by the fees levied from the DIS member institutions. Demirgüç-Kunt, Karacaovali and Laeven (2005) and other international surveys indicate that the majority of explicit deposit insurance systems are based on ex-ante funding. For such deposit insurance

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[1] The Subcommittee is headed by DIA Russia representative and includes representatives of deposit insurance agencies from Bulgaria, Colombia, Jordan, Kazakhstan, Korea, Mexico, the Philippines, Trinidad and Tobago, Turkey and USA.

[2], We would like to express our gratitude to representatives of deposit insurance agencies who have contributed to this paper: Dragomir Nedeltchev (Bulgaria), Andrez Florez (Colombia), Naser Salhi (Jordan), Lopez Zamora Guillermo (Mexico), Maria Leonida Fres-Felix (Philippines), Noel Nunes (Trinidad and Tobago), Rydvan Cabukel (Turkey).

systems, the problem of determining DIF sufficiency is similar to the problem of determining the economic capital adequacy of a lending institution over a given time horizon. Lending institutions accumulate loan loss reserves and maintain economic capital to offset the typical and extraordinary losses on a credit portfolio. Both loan loss reserves and economic capital are used as a safety cushion that comes to play when a lending institution faces a default on counterparty's liability. Likewise, the DIF is used in case of bankruptcy of a DIS member institution meaning that a lending institution is unable to meet its' liabilities to the holders of the insured deposits.

In order to estimate the sufficiency of the DIF, one should thus estimate the cumulative loss distribution (*CLD*) of the DIF, i.e. the function that maps outcomes (amounts of losses) to probabilities of these outcomes. From the *CLD*, one can calculate the expected loss (*EL*) and the unexpected loss (*UL*). The expected loss is the amount of loss one would expect to experience in a portfolio over the chosen time horizon. Formally, it is the typical value of the random variable with a given probability distribution, CLD, usually measured as the mean, median or mode value of the *CLD*. The unexpected loss measures the amount of risk in the portfolio. It is an estimate of atypically large losses that are rarely observable and therefore difficult to estimate statistically.

The desired level of the DIF is thus set to cover the expected loss (*EL*) and the unexpected loss (*UL*) of the DIS defined at a certain level of confidence over a given time horizon. The unexpected loss is usually defined as a quantile (percentile)[3] of the cumulative loss distribution (*CLD*). Alternatively, unexpected loss can be defined as a distance from the mean of the *CLD* measured in standard deviations. However, it is argued that this specification is less consistent (see Artzner et al, 1999).The desired level of the DIF is thus set to cover the expected loss (*EL*) and the unexpected loss (*UL*) of the DIS estimated at a certain level of confidence over a given time horizon. The confidence level for this percentile (i.e. the probability of the DIF deficit) can be set at the level corresponding to the probability of default on the sovereign debt of the home country of the DIS (see Bennett, 2002, Jarrow et al, 2005, and Smirnov et al, 2006).

[3] We recommend using median of CLD to measure EL, if UL is measured as quantile; mean loss can theoretically exceed given quantile. Additionally, Conditional Expected Loss (conditionally on the event of excess of given quantile) can be used.

Bennett (2002) remarks that a deposit insurer typically faces a skewed loss distribution: whereas there is a high probability of incurring small losses from a number of small lending institutions, there exists a low probability of incurring large losses in the unlikely event of failure of a large bank or a large number of smaller banks. Her evidence is further supported by empiric studies of Smirnov et al (2006). We can thus suggest that the standard deviation, which is routinely used as a universal risk measure, is insufficient to assess the adequacy of a deposit insurance fund, and other risk measures need to be considered.

The expected loss is usually defined as the sum over portfolio of insured lending institutions of individual exposures (E_i), times the estimated probability of default[4] (PD_i) times the expected loss given default (LGD_i):

$$EL = \sum_i EXP_i \cdot PD_i \cdot LGD_i$$

The existing models used by lending institutions as well as the Basel II Accord assume that the level of exposure is an exogenous input; quite a few relevant academic research is dedicated to modeling the exposure of a lending institution for the purpose of economic capital adequacy assessment. On the contrary, a wide array of publications over the last 50 years was dedicated to estimation of probabilities of default. In our survey, we outline the models currently employed (or developed) by the lending industry, as well as provide an outline of the relevant issues discussed in academic papers.

Commonly, deposit insurers are also claimants to the residual value of the failed lending institutions. For such deposit insurers recoveries can serve as a material source of funding, which makes accurate estimation of the recovery rates[5] an important task. Due to the lack of historical data, off-the-shelf models make strong simplifying assumptions concerning the cash flows generated in course of the recovery. However, recovery rate modeling has been receiving a growing attention from the academics over the last few years.

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[4] Assuming fixed maturities for all instruments; for different maturities, default intensity is a more relevant and flexible characteristic.

[5] Recovery rate (RR) is defined as: $RR = (1 - LGD)$.

The unexpected loss appears to be a more controversial issue. To estimate the potential extraordinary loss of the DIS, one should take into account the correlations between the probabilities of default of member institutions and also the cross-industry impact of macroeconomic shocks and the business cycle. Recent research suggests that significant correlations exist both across and between default intensities and recovery rates. More importantly, the default and recovery rates appear to change substantially in course of the business cycle, meaning that the macroeconomic and industrial environment should be accounted for when estimating these parameters and their interrelations. In our overview, we cover the practical approaches to unexpected losses estimation, as well as recent advances in joint estimation of *PD* and *LGD* and the relation of these parameters.

Summary of existing credit models

In business practice, proprietary and off-the-shelf credit models are used to derive the cumulative loss distribution using the standard building blocks outlined in the following section. In this section, we present a summary of these approaches to *CLD* estimation implemented in the industry accepted credit risk models: Credit Suisse's CreditRisk+, JPMorgan's CreditMetrics, McKinsey's CreditPortfolioView, and Moody's KMV's model. Individual building blocks of these models are covered in subsequent sections.

CreditRisk+ applies an actuarial approach to the derivation of the loss distribution. It is assumed that the probability distribution for the number of defaults is represented with a Poisson distribution. In turn, the average number of defaults per year, i.e. the sum of default probabilities over the portfolio, is represented with a Γ-distribution. Both *PD*s and *LGD*s are regarded as exogenous inputs. Apparently, this approach is only suitable for a homogenous portfolio of small individual obligors in a stable macroeconomic environment. The main advantage of this approach is that it offers a convenient analytical representation of the problem.

CreditMetrics methodology is based on the estimation of distribution of the changes in the credit portfolio value related to the migrations in credit quality of the obligors, including defaults, at a given time horizon.

The first step of the model is to specify the credit rating system together with the probabilities of migrating from one rating to another over the risk horizon. The model simplifies that all obligors within a given rating are homogenous with equal transition and default probabilities. Next, the forward discount rates are specified for each credit category and recovery rates are estimated for each category and/or seniority class. The correlations between changes in credit quality are derived from correlations of the obligors' assets. Monte Carlo simulation is then used to obtain the *CLD* function.

One can easily argue the adequacy of this model, as it relies heavily on historical averages that may be misleading. This modeling approach assumes that obligors within a given rating category are homogenous in term of migration and default probabilities. Further, both *PD*s and *LGD*s are estimated as historical averages, whereas it has been demonstrated in empiric literature that both parameters fluctuate widely through the business cycle.

CreditPortfolioView mimics the CreditMetrics approach, as it uses a migration matrix to assess the joint probabilities of default for the obligors. However, the default probabilities for each rating category and/or industry are estimated using a logit regression on a weighted index of macroeconomic factors. It is argued that in a diversified portfolio, idiosyncratic risks of individual obligors are efficiently diversified away, and only systematic risk of the macroeconomic environment needs to be considered. Further, the model also uses adjustment factors to account for the changes in the credit rating migration matrix due to the business cycle. It can thus be argued that this model provides a more adequate representation of the relation between credit risk and the business cycle. However, a strong assumption is that all obligors in the same industry and with the same credit rating have the same default probability given the state of the economy.

Unlike the models summarized above, KMV's model does not rely directly on historical average default frequencies and does not imply that the actual PDs equal these historical averages. Instead, the default probabilities, or expected default frequencies (*EDF*s) are derived for each obligor based on a Merton's (1974) type of model that relates the stochastic value of a firm's assets to the pre-determined value of its' liabilities. Asset value, business risk and leverage are combined into a single measure of default risk, called Distance-to-Default, which compares the market net worth to the size of one standard deviation move in the asset value. Moody's KMV uses actual default rates for companies in similar risk ranges to determine a functional relationship between Distance-to-Default and *EDF*. To calibrate the model, a large database of actual defaults is required.

The distinct feature of this approach is that is it essentially forward looking: under the efficient markets assumption, the market quotes of the firm's debt and equity incorporate all possible information regarding its' prospects, as well as the information on the industrial and macroeconomic environment. Additionally, *EDF*s can be updated in real time, whereas *PD*s estimated under other models rely on periodically disseminated macro- and microeconomic data, and credit rating changes tend to lag substantially behind the actual changes in credit quality.

The transition matrix constructed by KMV is based upon default rates rather than rating categories. Crouhy, Galai and Mark (2000) observe that the resultant table is characterized by lower default rates and higher transition probabilities than the credit migration table based on historical credit ratings data. KMV's approach to cash flow valuation is also materially different from those of the other firms. Given the term structure of *EDF*s for a given obligor, the net present value of a stream of contingent cash flows is derived under the martingale approach to securities pricing.

Similar to other models, KMV's model derives default correlations from a structural model that links correlations with fundamental factors. Finally, KMV provides an analytical derivation of the asymptotic loss distribution of the portfolio at a given time horizon, assuming the bank's loan portfolio is infinitely fine grained and that all instruments in the portfolio mature within this time horizon. This distribution is characterized by high skew and leptokurtosis.

1 Probability of default

Probability of default (*PD*) of member lending institutions is one of the key drivers of the overall risk of the deposit insurance system. There exist a variety of models deriving probabilities of default from various data sources.

The most intuitive approach is to bucket the obligors according to their aggregate level of risk proxied by independent, internal rating or supervisory rating, and then use the historical default frequencies that characterize the resulting buckets. However, lending industry firms are generally of high credit quality, meaning that default events are infrequent. Thus, consistent and robust *PD*s are hard to estimate from historical data. Also, the changing regulatory and market environment means that historical data on past defaults may be inappropriate to analyze (see Nickell, Perraudin and Varotto, 2000).

Another approach is credit scoring, which combines historical data on defaults with the financial and other relevant information on defaulted and non-defaulted obligors to derive an explicit equation for probability of default. These models are models are typically used for building credit risk models for retail banking and smaller corporate customers, where there are plenty of observations of defaulted companies. The shortcoming of these models is that, similarly to the historic default frequencies approach, they rely on abundance of relevant statistical data. Also, the complex nature of defaults means that probability of default demonstrates non-monotonous interrelation with the explanatory variables that are hard to estimate using conventional econometric techniques. Expert judgment needs to be employed to choose the best functional form of the econometric model.

For exchange-traded obligors, market models can be used to derive probabilities of default from the market prices of corporate debt and equity. The general disadvantage of these models is that they rely on the availability of market data and also on market efficiency (in particular, liquidity of the traded instruments). In the lending industry, however, most entities are either unlisted or thinly traded on the stock exchange, which restricts the use of such models for the purpose of *PD* estimation of DIS member institutions. Market models can be further categorized into structural models and reduced form models.

Structural models link the default of an entity to the value of the firm through its equity price. These models treat equity as an option to buy the company's assets, and use option pricing formulae to link the equity price, which is used as a proxy of the (generally unobservable) firm's asset value, to likelihood of default. The obvious benefit of such models is that they can use the latest market prices to provide a "marked to market" likelihood of default for individual companies. The major shortfall of structural models is that they deliberately simplify the capital structure of a firm, meaning that these models are hardly suitable for analyzing assets that have unusual capital structures or unusual pay-offs.

Gambler's ruin models can be regarded as a naïve practical application of structural models to non-traded firms. Assuming that a default occurs when a firm's net assets fall fellow a certain threshold (i.e. zero) and also assuming that firm's cash flows are independent and identically distributed, we can assess the probability of a lending institution's default over a given time horizon and also the expected time to default. However, the practical implementation of this theory to lending industry yielded mediocre results.

Reduced form models are generally used in areas such as bond and credit derivative pricing, and rather than producing likelihood of default measures, reduced-form models are generally used to calculate prices of such assets, or spreads between the assets' yields and the reference risk-free yield. Probabilities of default can then be derived from these spreads. The reduced-form models assume that the prices of such assets follow stochastic processes. Thus, there are two sets of difficulties: firstly the correct process has to be developed, and secondly the process needs calibrating to market data.

The latest academic thinking in the credit risk modeling area is that the debt and equity market data can be combined to produce hybrid models that allow for a more precise estimation of probabilities of default.

1.1 Credit ratings and historic default frequencies

The increasing availability of credit ratings assigned to corporate and sovereign debt issuers by independent rating agencies ensures the growing role of credit ratings in debt pricing. Chan-Lau (2006) distinguishes two ways of deriving *PD*s from independent credit ratings: these are cohort analysis and duration analysis. Cohort analysis is the simplest method to estimate default probabilities when credit ratings are available for a relatively large cross-section of firms or loans. For a given observation period, the probability of migrating from one credit rating to another is simply the observed proportion of firms that experience such migration. In particular, cohort analysis can be used to estimate the default probability given the credit rating of the firm or loan at the beginning of the period. Duration analysis accounts for the time spent in different credit ratings during the observation period. In duration analysis, the migration intensity is determined as the proportion of firm-years that migrated from one rating category to the other divided by the total number of firm-years.

Chan-Lau (2006) also comments on possible caveats of using rating-based credit risk models. First, actual default probabilities within a given credit rating bucket can vary widely, as credit ratings were developed to distinguish between high and low risk obligors, rather than to estimate precise *PD*s. Second, rating agencies consider several aspects of risk when assigning the credit ratings, and they may weight different criteria, such as default probability or loss given default, differently when assigning a rating.

Another caveat when using credit ratings is that they are constructed by factoring in expected business cycle conditions, a practice known as "through-the-cycle" ratings. Due to the difficulty of predicting the business cycle, the expected business cycle conditions are those corresponding to an average business cycle scenario. The downside of using an average business cycle scenario, however, is that ratings may not reflect reality well if the business cycle turns very differently from the average scenario used in the analysis. This fact underlies the criticism that ratings are too slow to react to news. In particular, Nickell, Perraudin, and Varotto (2000) observe that rating transition matrices, which capture the probability of migrating from one rating to another, are not stable through time since they depend on the stage of the business cycle.

Also, a problem specific for the lending industry is that in a stable environment, rated lending institutions carry a high credit rating, meaning that default events are rare. In such instance, historical data may be insufficient to provide consistent and robust *PD* estimates.

One way to account for lack of information in historical data is to incorporate expert opinions into the *PD* estimates. Practitioners, in particular those working in the emerging markets where historical data is scarce and the environment is rapidly changing, have long used expert estimates to assess the credit quality of the obligor. For instance, Pomazanov (2004) outlines an internal rating model where an expert's subjective judgment of the obligor's credit history and business risk account for 45% of the overall credit rating. However, until recently, no attempts were made to incorporate expert judgments into more formal quantitative models.

Kiefer (2006) argues that uncertainty about the default probability should be modeled the same way as uncertainty about defaults, namely, represented in a (prior) probability distribution for the default probability using Bayesian inference and expert information. The final (posterior) distribution should therefore reflect both data and expert information. Kiefer demonstrates that consistent expert opinions concerning the probabilities over probabilities in a portfolio of high-grade loans can be used to determine a four-parameter Beta distribution of probability of default. A number of questions are also raised concerning the efficient assessment and combination of expert information, robustness of the resultant distributions and implications for regulators suggesting further research into this area.

Another avenue of research suggests incorporating confidence intervals around estimated *PD*s into credit risk models. Schuermann and Hanson (2004) conduct a systematic comparison of confidence intervals around estimated probabilities of default using several analytical approaches from large-sample theory and bootstrapped small-sample confidence intervals over twenty-two years of credit ratings data. They find that the bootstrapped intervals for the duration-based estimates are much tighter when compared with the more commonly used (asymptotic) Wald interval. They also observe that even with these relatively tight confidence intervals, it is impossible to distinguish notch-level *PD*s for investment grade ratings—for example, a PD_{AA-} from a PD_{A+}. However, they are able to distinguish quite cleanly notch-level estimated default probabilities for obligors rated below investment quality. They also remark that conditioning on the state of the business cycle helps, as it is easier to distinguish adjacent *PD*s in recessions than in expansions.

Pluto and Tasche (2005) propose addressing the problem of low number of defaults with the most prudent estimation principle which, for each rating category, produces an upper bound or the most conservative estimate of the default probability. The principle imposes the constraint that default probabilities for a given rating category and those below it are the same, requiring that the ordinal ranking implied by the ratings is correct.

1.2 Theoretical framework

1.2.1 Credit scoring models

Credit scoring models are generally thought of as the ‘standard’ models for estimating the probability of default, and these models have also received wide recognition across the industry due to their technical simplicity. Basel II encourages the banks to use internal credit ratings to estimate their economic capital requirements; independent rating agencies use proprietary models to issue credit ratings to corporate borrowers; finally, deposit insurers use internal ratings to assess the risk level of the DIS member institutions and levy risk-adjusted deposit insurance premiums.

The fundamental idea of these models is to produce a credit score as a combination of the obligor’s quantitative parameter values and then map it to a probability of default based on historic default frequencies. The mapping can be performed by bucketing the obligors into several groups (credit grades or ratings) with known historic default frequencies, or by transforming credit scores into *PD*s using a logit/probit function.

1.2.1.1 Discriminant analysis

Discriminant function analysis is a statistical technique used to determine which variables discriminate between two or more pre-defined groups. Specifically, the method tests the statistical significance of the difference between the mean values of the parameter(s) in question between the groups. If the means for a variable are significantly different in different groups, then this variable discriminates between the groups. The more significant is the difference the better is the chosen parameter.

Financial ratios were in use to evaluate obligors' creditworthiness since the beginning of the 20th century. In his classical paper, Beaver (1966) was the first to provide formal analysis by conducting a comprehensive study using a variety of financial ratios. His conclusion was that the cash flow to debt ratio was the single best predictor. Altman (1968) used multiple discriminant analysis (MDA) to create a linear bankruptcy prediction model based on a limited number of financial ratios based on a stratified sample of 33 publicly-traded manufacturing bankrupt companies and matched them to 33 random non-bankrupt firms. The ratios used in the Altman model are: working capital over total assets; retained earning over total assets; earnings before interest and taxes over total assets; market value of equity over book value of total liabilities; and sales over total assets (Altman, 1968). The results yielded the well-known Z-score model that correctly classified 94% of the bankrupt companies and 97% of the non-bankrupt companies one year prior to bankruptcy. However, the results yielded two years prior to bankruptcy were far less impressive.

Altman, Haldeman and Narayanan (1977) addressed the issue by constructing the proprietary ZETA model that utilized a refined set of variables, as well as non-linear components. Altman (2002) extended his analysis to three larger samples from 1969-1999 covering different stages of the economic cycle. The model was between 82% and 94% accurate. He also extended the original Z-score model to include privately-held companies and privately-held non-manufacturing firms.

Although accrual accounting ratios were shown to predict bankruptcy accurately for the manufacturing industry, such financial ratios usually lack theoretical justification. Arguing that bankruptcy is first of all a cash flow event, Aziz and Lawson (1989) suggested a bankruptcy predictive model based on cash flow indicators. The proposed relation includes operating cash flow, net investment, liquidity change, taxes paid, shareholder cash flow, and lender cash flows. Comparing their model with Altman's updated analysis, Aziz and Lawson (1989) conclude that neither of the models is preferable over the whole period of observation.

Watson (1996) asserts that cash flow information does not contain any significant incremental information over the accrual accounting information to discriminate between bankrupt and non-bankrupt companies. He argues that cash flow from operations (CFFO) could be misleading because of management's manipulation of the timing of cash flows. Sharma (2001) claims that CFFO has not been properly measured; that some researchers did not validate their model; that cash flows and accrual data were highly correlated in the earlier days; and that incomplete information does not allow for study replication. Finally, Galai et al (2005) argue that even in the highly regulated banking environment, firms have the stimuli and the ability to produce smoothed earnings, which can have an adverse impact on the cash-flow based credit score estimates.

Grice and Dugan (2001) alert that problems can arise with models inappropriately applied. This could be the case when statistical models derived for a certain time period, industries, and financial distress situations are applied to situations other than those originally developed for. Their report evaluates the models developed by Zmijewski (1984) and Ohlson (1980). It is found that these models are sensitive to time periods. This means that the accuracy of the model decline when they are applied to time periods different from those used to develop and build the model. Furthermore, while Ohlson's model was sensitive to industry classification, Zmijjewski's model was not. However, neither model is sensitive to financial distress situations other than those used to develop the models.

The specifics of the banking industry mean that discriminant analysis yields significantly different results for lending institutions and for manufacturing firms. Moody's (2000) demonstrate that the distributions of private firms' financial ratios are substantially different from those of public firms, implying that these ratios have a different impact on private firms' *PD*s. Since most lending institutions are non-traded firms, discriminant analysis should not be performed for samples consisting of both private and exchange-traded firms.

Linear analysis was implemented by Stuhr and van Wicklin (1974) and Sinkey (1975) to discriminate between performing and nonperforming banks, where the status of the bank was determined by the supervisory body rather than by it filing a default. Altman (1977) loosened the equal dispersion matrix assumption to extend his multivariate analysis to the savings and loan associations. He developed a 12-variable econometric model combining three two-group discriminant models with quadratic parameters into a single rating that was reasonably efficient in predicting serious financial problems 1.5 years prior to the event over a sample of 56 serious problem, 49 temporary problem and 107 no problem savings and loan associations[7] over the period of 1966-1973. The parameters used included seven financial ratios, as well as two-year

[7]As defined by the Federal Savings and Loan Insurance Corporation of the USA (now merged with the FDIC).

trends of five key ratios. The overall in-sample classification accuracy of the model varied from 81% to 97% depending on the sub-sample analyzed.

1.2.1.2 Econometric models

Regression analysis of macro- and/or microeconomic time series is yet another credit scoring method. Macroeconomic-based models are motivated by the observation that default rates in the financial, corporate, and household sectors increase during recessions. This observation has led to the implementation of econometric models that attempt to explain default rates (and predict default probabilities) using economic variables. The econometric models can be further classified depending on whether they allow feedback between financial distress and the explanatory economic variables (autoregression). Wilson's (1998) CreditPortfolioView is a typical example of a macroeconomic econometric model applied to credit risk modeling. More recent examples may be found in Virolainen (2004) and Hoggarth, Sorensen and Zicchino (2005).

Chan-Lau (2006) underlines the disadvantages of econometric models based on macroeconomic time series. First, it is necessary that the data series span at least one business cycle; otherwise the model would not capture completely the impact of the business cycle on default probabilities. Second, under the rational expectations assumption the parameters and/or functional forms of such regressions are unlikely to remain stable. Finally, aggregate economic data are usually reported at substantial lags and are subject to revision, thus rendering macroeconomic-based models unsuitable for tracking rapidly deteriorating conditions of a firm or sector.

Hence, it appears that regressions based on microeconomic data are more suitable for credit risk analysis. In an early work, Martin (1977) runs a logit regression for a population of 5,700 Federal Reserve System member banks observed between 1970 and 1976, of which 58 banks were identified as failures. The failure/non-failure of a bank within two years after the observation of the explanatory variables was used as the dependent variable. The explanatory variables are drawn from a set of 25 financial ratios derived from the Report of Condition and Report of Income. These variables fall into four broad categories: asset risk; liquidity; capital adequacy; and earnings. The cut off point for determination of failed banks was set equal to the in-sample average default frequency. The model provided correct classification of 87% failed and 88.6% non-failed banks, meaning that 656 banks non-failed banks were classified as failed, and 3 failed banks were classified as non-failed. Overall, the degree of accuracy achieved by Martin is similar to that of the discriminant models of Altman (1968) and others.

More recent logit estimation of default probabilities of the US banks is provided in Estrella (2000) and Segoviano and Lowe (2002). The latter model is based on both microeconomic and macroeconomic variables. Golovan et al (2003 and 2004) use logit analysis to predict the default probabilities of Russian banks using the data between 1996 and 2001. They also demonstrate that clusterization of the initial sample based on the observed values of the explanatory variables results in superior forecasts. However, no robust tests are provided for cluster stability.

1.2.1.2.1 Hybrid models

Hybrid econometric models combine macroeconomic values, credit ratings, market variables and financial ratios to produce more accurate estimates of default probabilities.

Balzarotti, Falkenheim, and Powell (2002) propose an econometric model for estimating a loan's probability of default using historical data on a comprehensive set of loans originated in the Argentine financial system. Namely, they use an ordered probit specification where the explanatory variables are: the borrower's classification (or actual credit rating), the borrower's industrial activity classification, the size of the exposure, the CAMELS rating of the lender, and collateralization ratio of the loan.

Jiménez and Saurina (2005) studied the impact of rapid credit growth on loan losses in Spain by constructing a logit regression of *PD* on lagged characteristics of the loan (size, maturity, and collateral), regional dummies and industrial dummies, as well as originating bank characteristics, originating bank's loan portfolio growth rate, GDP growth and real interest rate. Their research confirmed the statistical significance of loan portfolio growth rate, implying for further research in the rapidly growing emerging markets.

1.2.1.3 Significance of non-monotonous relations

Altman (1977) recognizes the importance of non-monotonous relation between the probability of default and the financial ratios by including quadratic explanatory variables in his discriminant model. Further research by Smirnov et al (2006) confirms that quadratic approximation may be inappropriate to capture the actual relation between the financial ratios and default probabilities. Studying a sample of 18527 random observations (including 593 observations of default) over a total of 1663 banks between years 1993 and 2005, the researches observe that the non-monotony of this relation cannot be adequately captured by polynomial models. Instead, Smirnov at al (2006) perform a discretization of the explanatory variables using up to three threshold values, which allows for more accurate estimation of probabilities of default. Obviously, other nonlinear approaches, such as neural networks, can be applied to solve the problem (for example, see Wilson and Sharda, 1994). Dwyer, Kocagil and Stein (2004) also mention that non-parametric transforms of financial ratios are implemented in Moody's KMV RiskCalc v3.1 model.

1.2.2 Gambler's ruin models

Gambler's ruin models can be perceived as an early and somewhat less successful structural approach to credit risk modeling that do not rely on market data. These models assume that a firm's default occurs when the book value of its' net assets fall below a certain threshold level, typically set at zero. The parameters of the stochastic process of net asset value are estimated using the historic data. The distinctive feature of these models is that they were developed specifically for the lending industry, where frequent observations of net asset value of a wide universe of institutions were available from the industry regulator. The advantage of these models is that they do not rely on market data input and can thus be applied to non-traded lending institutions. However, these models have proven to be poorly fitted to the empiric data.

Wilcox (1976), Santomero and Vinso (1977), and Vinso (1979) pioneered the bankruptcy prediction models based on the gambler's ruin model of probability theory. In these models, the firm is viewed as a gambler who begins the game with an amount of money equal to its net assets. It wins an incremental amount of net assets with probability p or loses it with probability ($1-p$) and bankruptcy occurs when the firm's net worth falls to zero. The dynamics of a firm's net assets can thus be described by a stochastic process estimated on the time series of net assets[8].

The attempts to apply this model have been disappointing, perhaps because the version of the theory used is too simple, assuming, as it does, that cash flow results from a series of independent trials, without the benefit of any intervening management action. Although the theory specified a functional form for the probability of ultimate ruin, Wilcox (1976) found that this probability was not meaningful empirically. It could not be calculated for over half of his sample because the data violated the assumptions of the theory: firms that were supposed to be bankrupt were actually solvent. Faced with this problem, Wilcox (1976) discarded the functional form suggested by the theory and used the variables it suggested to construct a prediction model. It is hard to assess the resulting model since he did not test it on a holdout sample.

[8] Such data on US banks was available on a weekly basis from the Federal Reserve System of the USA in the 1970s.

Santomero and Vinso (1977) provide an empirical application of a gambler's ruin-type model using bank data. However, they provide no test of the model. Their approach is more complex than that of Wilcox, in that they estimate the probability of failure for each bank at that future point in time when its probability of failure will be at a maximum. They estimate the probability of failure for the median bank in their sample will reach a maximum of 1×10^{-99} at a point in time 35 years hence. In addition, the riskiest bank in their sample may have a probability of failure as low as 0.0000003. Considering the history of bank failure, these probabilities are implausibly low. Vinso (1979) uses a version of the gambler's ruin model to estimate default probabilities. However no rigorous tests are presented.

An extension of the gambler's ruin problem consistent with the Merton model in the assumption that a firm's book equity is not equivalent to its' value, was proposed by Scott (1981). His adaptation recognizes the fact that companies go bankrupt not because they run out of cash, but rather because of loosing public confidence.

1.2.3 Market-based models

1.2.3.1 Structural models

Merton (1974) was the first to use the principles of option pricing developed by Black and Scholes (1973) to create a structural credit risk model framework. In such a framework, the default process of a company is driven by the value of the company's assets and the risk of a firm's default is therefore explicitly linked to the variability of the firm's asset value. The basic intuition behind the Merton model is relatively simple: default occurs when the value of a firm's assets (the market value of the firm) is lower than that of its liabilities. The payment to the debt holders at the maturity of the debt is therefore the smaller of two quantities: the face value of the debt or the market value of the firm's assets. Assuming that the company's debt is entirely represented by a zero-coupon bond, if the value of the firm at maturity is greater than the face value of the bond, then the bondholder gets back the face value of the bond. However, if the value of the firm is less than the face value of the bond, the shareholders get nothing and the bondholder gets back the market value of the firm. The payoff at maturity to the bondholder is therefore equivalent to the face value of the bond minus a put option on the value of the firm, with a strike price equal to the face value of the bond and a maturity equal to the maturity of the bond. Following this basic intuition, Merton derived an explicit formula for risky bonds which can be used both to estimate the *PD* of a firm and to estimate the yield differential between a risky bond and a default-free bond.

In addition to Merton (1974), first generation structural-form models include Black and Cox (1976), Galai, and Masulis (1976), Geske (1977), and Vasicek (1984). Each of these models tries to refine the original Merton framework by removing one or more of the unrealistic assumptions. Black and Cox (1976) introduce the possibility of more complex capital structures, with subordinated debt; Geske (1977) introduces interest-paying debt; Vasicek (1984) introduces the distinction between short and long term liabilities which now represents a distinctive feature of the KMV model. The KMV model assumes that default occurs when the firm's asset value goes below a threshold represented by the sum of the total amount of short term liabilities and half of the amount of long term liabilities

Although the line of research that followed the Merton approach has proven very useful in addressing the qualitatively important aspects of pricing credit risks, it has been less successful in practical applications. This lack of success has been attributed to different reasons. First, under Merton's model the firm defaults only at maturity of the debt, a scenario that is at odds with reality. Second, for the model to be used in valuing default-risky debts of a firm with more than one class of debt in its capital structure (complex capital structures), the priority/seniority structures of various debts have to be specified. Also, this framework assumes that the absolute-priority rules are actually adhered to upon default in that debts are paid off in the order of their seniority. However, empirical evidence collected by Franks and Torous (1994), indicates that the absolute-priority rules are often violated. Moreover, the use of a lognormal distribution in the basic Merton model (instead of a more fat tailed distribution) tends to overstate recovery rates in the event of default.

In response to such difficulties, an alternative approach has been developed which still adopts the original Merton framework as far as the default process is concerned but, at the same time, removes one of the unrealistic assumptions of the Merton model**;** namely, that default can occur only at maturity of the debt when the firm's assets are no longer sufficient to cover debt obligations. Instead, it is assumed that default may occur anytime between the issuance and maturity of the debt and that default is triggered when the value of the firm's assets reaches a lower threshold level. These models include Kim, Ramaswamy and Sundaresan (1993), Hull and White (1995), Nielsen, Saà-Requejo, and Santa Clara (1993), Longstaff and Schwartz (1995) and others.

Despite these improvements with respect to the original Merton's framework, second generation structural-form models still suffer from three main drawbacks, which represent the main reasons behind their relatively poor empirical performance (Eom, Helwege and Huang, 2001). First, they still require estimates for the parameters of the firm's asset value, which is non-observable. Indeed, unlike the stock price in the Black and Scholes formula for valuing equity options, the current market value of a firm is not easily observable. Second, structural-form models cannot incorporate credit-rating changes that occur quite frequently for default-risky corporate debts. Most corporate bonds undergo credit downgrades before they actually default. As a consequence, any credit risk model should take into account the uncertainty associated with credit rating changes as well as the uncertainty concerning default. Finally, most structural-form models assume that the value of the firm is continuous in time. Duffie and Lando (2001) argue that as a result, the time of default can be predicted just before it happens and hence, there are no sudden defaults. In other words, without recurring to a "jump process", the *PD* of a firm is known with certainty.

1.2.3.2 Reduced-form models

The attempt to overcome the above mentioned shortcomings of structural-form models gave rise to reduced-form models. These include Litterman and Iben (1991), Madan and Unal (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1998), Duffie (1998), and Duffie and Singleton (1999). Unlike structural-form models, reduced-form models do not condition default on the value of the firm, and parameters related to the firm's value need not be estimated to implement them. In addition to that, reduced-form models introduce separate explicit assumptions on the dynamic *PD*, which is modeled independently from the structural features of the firm, its asset volatility and leverage. Reduced-form models fundamentally differ from typical structural-form models in the degree of predictability of the default as they can accommodate defaults that are sudden surprises. A typical reduced-form model assumes that an exogenous random variable drives default and that the probability of default over any time interval is nonzero. Default occurs when the random variable undergoes a discrete shift in its level. These models treat defaults as unpredictable Poisson events. The time at which the discrete shift will occur cannot be foretold on the basis of information available today.

To apply reduced-form models in practice, it is important to determine credit spreads for a group of firms with different credit risk qualities correctly. A general methodology and bibliography can be found in Smirnov et al (2006b), adopted as a standard by the European Bond Comission (EFFAS-EBC) in 2006. This methodology was originally conceived for Eurozone government bond market, but it is applicable to domestic markets as well. In particular, in Smirnov et al (2006b) the approach was successfully tested for the Swiss municipal bond market.

Empirical evidence concerning reduced-form models is rather limited. Using the Duffie and Singleton (1999) framework, Duffee (1999) finds that these models have difficulty in explaining the observed term structure of credit spreads across firms of different credit risk qualities. In particular, such models have difficulty generating both relatively flat yield spreads when firms have low credit risk and steeper yield spreads when firms have higher credit risk.

It is important to understand that reduced form models deal with risk-neutral default intensities rather than real-world intensities. Generally speaking, default rates inferred from credit spreads are risk-neutral estimates that include a risk premium, whereas default rates estimated by the econometric model are real-world estimates that do not include a risk premium, since they are based on historical data. The empirical literature in this area often interprets the drift adjustments of the default intensity's diffusion state variables as the only default risk premium. In Merton style structural models calibration is often performed using CAPM, so that the mapping between risk-neutral and real-world default rates is defined by one parameter, the Sharpe ratio.

Jarrow et al (2005) point out that this interpretation implies a restriction on the form of the possible default risk premia, which can be justified through exact and approximate notions of "diversifiable default risk." Under conditional diversification condition, the actual and risk-neutral default intensities coincide, which greatly facilitates the pricing and management of credit risk. It does not mean, however, that actual and risk neutral survival probabilities are the same (see Duffie and Singleton, 2003). Therefore, risk-neutral default rates extracted from bond spreads contain both liquidity premium and risk premium and can be considered as upper bounds for actual probabilities.

1.2.3.3 Reconciliation models

A number of recent research papers is dedicated to reconciliation of structural and reduced form models based on the incomplete information assumption. Schönbucher (1996) presents way of bridging the gap between structural and reduced models based on the introduction of a jump in the firm's asset value process. However, Duffie and Lando (2001) argue that this approach, although solves the problem of zero short term spreads in structural models, does not theoretically bridge the gap between structural and reduced form models because it is not (generally) consistent with a default intensity. Instead, Duffie and Lando (2001) consider a model in which the default time is fixed by the firm's managers in order to maximize the value of the equity, as in Leland and Toft (1996), considering a geometric Brownian motion for the asset process. However, investors cannot observe the issuer's assets directly, and receive only periodic and imperfect accounting reports. Duffie and Lando (2001) derive the distribution of the firm's asset value conditional to investors' information and from it the intensity of default in terms of the conditional asset distribution and the default threshold.

Çetin et al. (2004) propose a similar approach for linking structural and intensity models to the one by Duffie and Lando (2001), assuming that investors receive only a reduced version of the information that firm's managers have available. They claim that the default time is a predictable event for firm's managers, since they have enough information about the firm's fundamentals, however public investors do not have access to that information. In their model, the firm's cash flow is the variable which triggers default, after reaching some minimum levels during a given period of time. Firm's managers can see the cash flow levels, but investors only receive information about the sign of the cash flow, making the default time an unpredictable event for them. In this setting, they derive the default intensity as seen by the market. As Duffie and Lando (2001), they do not make use of neither the compensator nor the pricing trend process.

Giesecke (2004) takes the incomplete information assumption in structural model one step further: the modeling of default correlation. He provides a structural model in which the firms' default probabilities are linked via a joint distribution for their default thresholds. Investors do not have perfect information about neither such thresholds nor about their joint distribution. However, they form a prior distribution which is updated at any time one of such thresholds is revealed, which only happens when one of the firms defaults.

In Giesecke (2004) investors have incomplete information about the firms' default thresholds but complete information about their asset processes. Giesecke and Goldberg (2004b) extend that framework to one in which investors do not have information neither about the firms' asset values nor about their default thresholds. In this case, default correlation is introduced through correlated asset processes and, again, investors receive information about the firms' asset and default barrier only when they default. Such information is used to update their priors about the distribution of the remaining firms' asset values.

Giesecke and Goldberg (2004a) consider the case in which the default barrier is random and unobserved, which they argue is consistent with recent experiences at Enron, WorldCom, and Tyco, which surely show that investors cannot observe the barrier. In each of these cases, the true level of liabilities was not disclosed to the public. Not knowing the default threshold, investors use a priory distribution for its value.

Giesecke (2005) deals first with the case of a structural model in which investors have complete information about both the level of the firm's asset value and the default threshold. Considering a continuous process for the asset value, we are in the case of a standard first passage model which implies predictable defaults. After that, he deals with the case of complete information about the asset value but incomplete information about the default threshold. Although constant, the default threshold is not known by the investors, who are forced to work under a distribution function for the default threshold. The impossibility of observing the default threshold makes the default time an unpredictable event. In this case, investors can calculate the pricing trend in terms of the distribution function for the threshold and the observable historical asset value. However, in this specification of the information setting, the pricing trend does not admit an intensity of default. Next, Giesecke (2005) studies the cases of incomplete information for the asset value (instead of the default threshold) and for both the asset value and the default threshold. In contrast with the previous case, the pricing trend calculated in terms of the threshold distribution and the distribution for the minimum historical asset level, admits an intensity representation.

Guo, Jarrow and Zeng (2005) consider that the ways in which the previous papers introduce incomplete information about the variables generating default are illustrative but too simple to be applied in practice. Their paper represents an extension and generalization of the previous models, in an attempt to formalize the theory linking reduced and structural models.

1.2.4 Hybrid models

Hybrid credit risk models are a recent advancement of the credit risk modeling that combines credit scoring techniques and market based models to produce superior estimates of default probabilities. Shumway (2001) claims that the scoring models of Altman (1968), Altman, Haldeman, and Narayanan (1977), Ohlson (1980), and Zmijewski (1984) are misspecified, and suggests an alternative hazard model to account for time impact. The proposed model uses a combination of accounting ratios, such as the ratio of net income to total assets and the ratio of total liabilities to total assets, and market-driven variables, such as market size, past stock returns, and the idiosyncratic standard deviation of stock returns to produce a more accurate out-of-sample forecast of *PD*s than traditional econometric models.

Hazard models resolve the problems of static models by explicitly accounting for time. The dependent variable in a hazard model is the time spent by a firm in the healthy group. When firms leave the healthy group for some reason other than bankruptcy (e.g., merger), they are considered censored, or no longer observed. The hazard model can be thought of as a binary logit model that includes each firm-year as a separate observation. Detailed description of hazard models can be found in Kiefer (1988) and Lancaster (1990).

Shumway (2001) points out that hazard models allow controlling for each firm's period at risk. Indeed, some firms file for bankruptcy after many years of being at risk while other firms fail in their first year. Static models do not adjust for period at risk, but hazard models adjust for it automatically. Next, hazard models incorporate time-varying co-variates, or explanatory variables that change with time. If a firm deteriorates before bankruptcy, then allowing its financial data to reveal its changing health is important. Hazard models exploit each firm's time-series data by including annual observations as time-varying covariates. Unlike static models, they can incorporate macroeconomic variables that are the same for all firms at a given point of time. Hazard models can also account for potential duration dependence, or the possibility that firm age might be an important explanatory variable. Finally, hazard models allow for utilizing much more data. Shumway (2001) claims that all of these results in more precise estimates and superior forecasts delivered by his model.

Analyzing the data on exchange-traded non-financial firms that began trading between 1962 and 1992, Shumway (2001) finds that while half of the accounting ratios used previously are poor predictors, several previously neglected market-driven variables are strongly related to bankruptcy probability. He also demonstrates that the hazard model outperforms the traditional discriminant analysis models out of sample and produces results that are quite accurate.

Chava and Jarrow (2001) study a sample of 167,617 observations over 17,460 firms (including 1,197 bankruptcies) in 1962-1999 to validate the superior performance of Shumway's (2001) model in comparison with traditional scoring models. They further discover that the use of monthly observations ensures a significant boost of the model's efficiency. The authors further compare 'private firm' and 'public firm' specifications of Shumway's model. The private firm specification is built on accounting values only, whereas the public firm specification uses just the market variables. The public firm model was demonstrated to be superior to the private firm model, with the accounting values marginally contributing to the accuracy of the generalized model. This is consistent with an efficient market hypothesis stating that market prices reflect all publicly available information regarding bankruptcy, including that contained in the accounting variables. Also, industry effects represented by dummy variables were demonstrated to be statistically significant for bankruptcy prediction.

Comparing the models estimated on the samples of financial and non-financial corporations, Chava and Jarrow (2001) conclude that the hypothesis of equal predictive power cannot be rejected. They also carry out a comparison of Shumway's hazard rate model and a reduced form model specified in Janosi, Jarrow and Yildirim (2000) to conclude that the hypothesis that the explicit default intensities (hazard rates) and implicit default intensities estimated with the reduced form model are equal cannot be rejected.

The approach outlined by Chava and Jarrow (2001) found a practical implementation in Kamakura's Risk Manager model.

1.3 Commercial applications

J.P. Morgan's CreditMetrics (Gupton, Finger and Bhatia, 1997) relies on exogenous specification of a rating system and a transition matrix describing the probabilities of migrating from one credit rating to another. A strong assumption made by the model is that all issuers are credit-homogeneous within the same rating class, with the same transition probabilities and the same default probability that can be estimated based on historical data. The probability of default for a given obligor at a given time horizon is thus defined as the probability of the obligor migrating from its current credit rating into default.

Carty and Liebermann (1996) demonstrate that the default frequencies vary in time, partly in reflection of the business cycle, meaning that backward-looking historical averages can be misleading, and the variability of the default frequencies needs to be incorporated into default probability model. More importantly for the deposit insurance, not all lending institutions are independently rated, meaning that supervisory ratings may be used instead. More importantly, the insufficiency and irrelevance of historical data due to the rarity of failure of insured lending institutions may further undermine the efficiency of CreditMetrics-styled models. Empiric studies (e.g. Smirnov et al, 2006) demonstrate that rating adjustments typically lag behind the changes in credit quality, rather than reflect the current credit quality of an obligor.

CreditRisk+ (CSFB, 1997) promotes an actuarial approach to credit risk modeling. The model assumes that defaults are infrequent and independent from each other, and default probabilities for all obligors are equal. In order to better fit the empiric data, it is assumed that the default probability is a random variable characterized by a gamma distribution. The parameters of this distribution can be estimated based on historic data. The volatility of the default probability thus accounts for the influence of default correlation and background factors, such as a change in the rate of growth in the economy which may in turn affect the correlation of default events.

It can easily be argued that a deposit insurer's portfolio is not diversified enough, and the actual probabilities of default of insured lending institutions vary widely, thus making the CreditRisk+ approach inapplicable to deposit insurance.

McKinsey's CreditPortfolioView (Wilson, 1998) implements an econometric model of default probability dependent on macroeconomic variables, such as the unemployment rate, the rate of growth in GDP, the level of long-term interest rates, foreign exchange rates, government expenditures, and the aggregate savings rate. Default probabilities are modeled as a logit function where the explanatory variable is a country or industry-specific composite index. The index represents the level of systematic risk and is estimated as a linear combination of current and lagged macroeconomic variables. The use of a single systemic risk index is justified by the consideration that in a well diversified portfolio all idiosyncratic risks are diversified away, leaving the lender to deal with the systemic risk only. In the proposed implementation, each macroeconomics variable is assumed to follow a univariate, auto-regressive model of order 2 with independent and identically distributed errors.

Admittedly, a deposit insurer's portfolio dos not meet the prerequisites of Wilson's (1998) model: all contingent liabilities originate from the same country and from the same industry, meaning that idiosyncratic risks are of paramount importance. Moreover, it is observed that in case of systematic crisis blanket guarantee can be provided by the government, implying that the DIS is not intended to protect against systemic risk (Demirgüç-Kunt and Detragiache, 1999).

KMV Corporation, purchased by Moody's in 2002, has created a procedure for estimating the default probability of a firm that is based conceptually on Merton's (1974) option-theoretic, zero coupon corporate bond valuation approach. In three steps, it determines an expected default frequency for a company.

In the first step, the market value and volatility of the firm's assets are estimated. In the contingent claim approach to the pricing of corporate securities,. the log-asset returns follows a normal distribution, with the returns being independent for non-overlapping time intervals and homogeneous (that is, the volatility of the firm's market value remains relatively stable over time). Since only the market value of a firm's assets and traded debt is observable, the model employs the option pricing model to the estimate the value of corporate liabilities as suggested in Merton (1974), and further elaborated in Crouhy and Galai (1994), Bensoussan et al. (1994, 1995), and Fong and Vasicek (1997). In order to make the model tractable, KMV assumes that the capital structure is only composed of equity, short-term debt which is considered equivalent to cash, long-term debt which is assumed to be perpetuity, and convertible preferred shares. The value of assets can then be estimated as a function of the observable values of assets, leverage ratio, average coupon yield on the long-term debt; and unobservable value of the volatility of asset returns. An iterative technique is then used to calibrate the model for volatility of asset returns.

In the second step, the distance to default is calculated. In the option pricing framework, default, or equivalently bankruptcy, occurs when assets value falls below the value of the firm's liabilities. In practice, default is distinct from bankruptcy which corresponds to the situation where the firm is liquidated, and the proceeds from the assets sale is distributed to the various claim holders according to pre-specified priority rules. Default is the event when a firm misses a payment on a coupon and/or the reimbursement of principal at debt maturity. KMV has observed from a sample of several hundred companies that firms default when the asset value reaches a level somewhere between the value of total liabilities and the value of short-term debt. For all these reasons, KMV implements an intermediate phase before computing the probabilities of default: it computes distance to default as the number of standard deviations between the mean of the distribution of the asset value, and a critical threshold, the 'default point', which is set at the par value of current liabilities including short term debt to be serviced over the time horizon, plus half the long-term debt.

This last phase consists of scaling the *DD*s to the actual probabilities of default (*EDF*s), for a given time horizon, using the historical data on *EDF*s and respective defaults.

The obvious advantage of this model is that it relies on market prices. Provided that the market is efficient, the prices incorporate all available information regarding the future events, thus making market a better predictor than the backward-looking macro- or microeconomic statistics. The other side of the coin is that the model cannot be applied to non-traded or thinly-traded firms that are abundant in the lending industry. To address this issue, Dwyer, Kocagil and Stein (2004) developed Moody's KMV RiskCalc v3.1 model as an off-the-shelf default probability model for private firms. At the core of the model is a probit function of non-parametric transforms of financial performance indicators falling under seven broad categories: profitability, leverage, debt coverage, growth variables, liquidity, business activity, and size. The exact selection of indicators is specific for each lending environment. RiskCalc model also incorporates market data by using the sector's average distance-to-default as one of the explanatory variables in the private firms' *EDF* equation. Authors also claim that when the distance-to-default measure is imbedded into the model, macroeconomic variables (such as interest rate, GDP growth or unemployment rate) do not contribute to the explanatory power of the model. Intuitively, the market prices that are used to calculate the average distance-to-default already incorporate all available macroeconomic data, given the informational efficiency of the market. Moreover, average distance-to-default allows incorporating the industry-wide trends into the model, thus allowing for more precise estimates of default probabilities.

It can easily be argued that the suggested approach cannot be considered as a well suitable tool for the lending industry and hence for deposit insurance applications, since all contingent liabilities in a deposit insurer's portfolio belong to the same industry. Also, since all liabilities are also within the same national economy, it can be suggested that macroeconomic variables do not need to be incorporated into a probability of default model. Instead, a single variable characterizing the stage of the business cycle can be used. Finally, it can be observed that it is generally the few major lending institutions that are actively traded, with the substantial impact of size effect meaning that the distance-to-default for such banks will be of little significance for the default probability of the median lending institution.

In recognition of the specific nature of the lending industry, Sellers and Arora (2004) suggest modeling a financial firm as a combination of two businesses: holders of a portfolio of financial claims, and providers of business services. The researchers argue that this dual business nature has direct implications for the *EDF* measures of some of these large financial firms.

During good times, the incremental gain in the firm's market asset value comes from riskier businesses like intermediation, advisory, and servicing. The high volatility of these businesses and the fact that they are a larger proportion of the total business lead to an overall increase in the firm's asset volatility, reducing or reversing the positive impact on the credit quality that is coming from increased asset value. During bad times, these firms lose some of these riskier business opportunities, and shed some of that extra volatility. The authors argue that the volatility of the riskier franchise business can be 10 times as high as the volatility of the asset portfolio, having a substantial effect on the volatility of the overall firm value.

Sellers and Arora (2004) further argue that the asset portfolio of a financial institution is reflected by its book value, whereas the expectations of its non-portfolio earnings are captured by its market value. The difference between the market value and the book value can thus proxy the value of the franchise business. Assuming perfect correlation of portfolio and non-portfolio components, volatility of the non-portfolio value can be estimated by a simple regression. Long-term portfolio and franchise components of the firm value can then be estimated to assess the normalized volatility of the financial firm value and estimate the *EDF* based on its value.

However, empiric comparison carried out by Sellers and Arora (2004) for the observations between 1996 and 2004 demonstrates that the refined model contributes less than 1% improvement to the accuracy ratio of *EDF* estimates.

2 Loss Given Default

Basel II Capital Adequacy accord and most of the industrial credit risk models use loss given default[9] (*LGD*) as one of the building blocks for estimation of the expected loss of a credit portfolio, defining loss given default as the ratio of losses to exposure at default.

2.1 Measuring loss given default

Shuermann (2004) discerns three approaches to *LGD* estimation.

- *Market LGD* is arguably the easiest way to estimate the recoveries. It can be observed from market prices of defaulted bonds or marketable loans soon after the actual default event. Rating agencies carry out regular recovery studies based on this approach[10]. These prices have some desirable properties since they are observed early and are a reflection of market sentiment at that time, thus incorporating the expert opinions of all market participants regarding the expected recovery, suitably discounted, and thus include recoveries on both discounted principal and missed interest payments, as well as restructuring costs and uncertainty of that restructuring process.
- *Workout LGD* is somewhat more ambiguous. It can be estimated as the ratio of projected cash flows resulting from the workout and/or collections process, discounted at a relevant rate, to the estimated exposure. Attention needs to be paid to the timing of the cash flows from the distressed asset. Measuring this timing will impact downstream estimates of realized *LGD*. The cash flows should be discounted, but it is by no means obvious which discount rate to apply. For example, the debt restructuring could result in the issuance of risky assets such as equity or warrants, or less risky ones such as notes, bonds or even cash. In principle the correct rate would be for an asset of similar risk. Importantly, once the obligor has defaulted, the bank is an investor in a defaulted asset and should value it accordingly.

[9] Or recovery rate (*RR*): *RR* = (1-*LGD*)

[10] For instance, Moody's observes the market prices of defaulted bonds one month after the first occurrence of the default event

- *Implied market LGD* can be derived from risky bond prices using some theoretical asset pricing model. The advantage of this approach is that the universe of non-defaulted bonds exceeds the universe of defaulted instruments, meaning that much more information is available for analysis. However, a debt instrument's risk premium reflects expected loss, that needs to be decomposed into *PD* and *LGD*, as well as liquidity premium. Bakshi, Madan and Zhang (2001) and Unal, Madan and Guntay (2003) suggest techniques for decomposing the bond spreads and deriving *LGD* measures.

2.2 Modeling loss given default

2.2.1 Theoretical framework

2.2.1.1 Structural models

Under these models**,** all the relevant credit risk elements, including default probabilities and recovery at default, are a function of the structural characteristics of the firm: asset levels, asset volatility (business risk) and leverage (financial risk). The *RR* is therefore an endogenous variable, as the creditors' payoff is a function of the residual value of the defaulted company's assets. More precisely, under Merton's theoretical framework, *PD* and *RR* tend to be inversely related. If, for example, the firm's value increases, then its *PD* tends to decrease while the expected *RR* at default increases (ceteris paribus). On the other side, if the firm's debt increases, its *PD* increases while the expected *RR* at default decreases. Finally, if the firm's asset volatility increases, its *PD* increases while the expected *RR* at default decreases, since the possible asset values can be quite low relative to liability levels.

Under the 'second generation' structural models**,** the *RR* in the event of default is exogenous and independent from the firm's asset value. It is generally defined as a fixed ratio of the outstanding debt value and is therefore independent from the *PD*. For example, Longstaff and Schwartz (1995) argue that, by looking at the history of defaults and the recovery rates for various classes of debt of comparable firms, one can form a reliable estimate of the *RR*. In their model, they allow for a stochastic term structure of interest rates and for some correlation between defaults and interest rates. They find that this correlation has a significant effect on the properties of the credit spread[4]. This approach simplifies the first class of models by both exogenously specifying the cash flows to risky debt in the event of bankruptcy and simplifying the bankruptcy process. The latter occurs when the value of the firm's underlying assets hits some exogenously specified boundary.

2.2.1.2 Reduced-form models

Generally speaking, reduced-form models assume an exogenous *RR* that is independent from the *PD* and the dynamics of a firm's assets, and take as basics the behavior of default-free interest rates[11], the *RR* of defaultable bonds at default, as well as a stochastic process for default intensity. At each instant, there is some probability that a firm defaults on its obligations. Both this probability and the *RR* in the event of default may vary stochastically through time. Those stochastic processes determine the price of credit risk. Although these processes are not formally linked to the firm's asset value, there is presumably some underlying relation.

Reduced-form models somewhat differ from each other by the manner in which the *RR* is parameterized. For example, Jarrow and Turnbull (1995) assumed that, at default, a bond would have a market value equal to an exogenously specified fraction of an otherwise equivalent default-free bond. Duffie and Singleton (1999) followed with a model that, when market value at default is exogenously specified, allows for closed-form solutions for the term-structure of credit spreads. Their model also allows for a random *RR* that depends on the pre-default value of the bond. While this model assumes an exogenous process for the expected loss at default, meaning that the *RR* does not depend on the value of the defaultable claim, it allows for correlation between the default hazard-rate process and *RR*. Indeed, in this model, the behavior of both *PD* and *RR* may be allowed to depend on firm-specific or macroeconomic variables and therefore to be correlated.

Other models assume that bonds of the same issuer, seniority, and face value have the same *RR* at default, regardless of the remaining maturity. For example, Duffie (1998) assumes that, at default, the holder of a bond of given face value receives a fixed payment, irrespective of the coupon level or maturity, and the same fraction of face value as any other bond of the same seniority. This allows him to use recovery parameters based on statistics provided by rating agencies such as Moody's. Jarrow, Lando and Turnbull (1997) also allow for different debt seniorities to translate into different *RR*s for a given firm. Both Lando (1998) and Jarrow, Lando and Turnbull (1997) use transition matrices to price defaultable bonds.

[11] The stochastic evolution for term structure of interest rates can be easily incorporated in the model in the framework of reduced form model; that is one of the major advantages of the approach.

2.2.2 Empiric studies

Independent rating agencies provide average recovery rates based on historical data. Basel II Capital Accord fundamental approach also assigns average *LGD*s to various asset classes. Although it may be tempting to use the average estimates, Schuermann (2004) observes that recovery rates tend to be either high (70%-80%) or low (20%-30%), thus producing a bimodal distribution of recovery rates. Thus, average *LGD* estimations may be misleading, and more precise modeling is required.

Altman and Kishore (1996) demonstrate that seniority of the debt is one of the primary drivers of *LGD*. These findings are also supported by Acharya, Bharath and Srinivasan (2006). Gupton, Gates and Carty (2000) estimate the average recovery rate for senior secured and unsecured debt at 70% and 52%, respectively. This implies that loans should have lower *LGD*s than bonds; however, little information is publicly available on loan recovery rates. Amihud, Garbade and Kahan (2000) also point out that loans better control the agency costs of debt through tighter covenants, renegotiation, and closer monitoring, which is another argument in favor of higher loan recoveries. However, Eberhart and Weiss (1998) remark that in practice priority rules are often violated, as the bondholders are ready to dismiss their priority in favor of a more speedy resolution.

Apparently, original credit rating of a bond issue has virtually no effect on recoveries once seniority is accounted for. Neither the time to default from original date of issuance, nor issue size has any substantial impact on the recoveries (Altman and Kishore, 1996; Carty and Lieberman, 1996). On the contrary, the sectoral differences are statistically significant even when accounting for seniority. Studying a sample of 696 defaults between 1978 and 1995, Altman and Kishore (1996) demonstrate that public utilities and chemicals and petroleum industry demonstrate the highest average recovery rates (70% and 63%, respectively) compared with the overall average (58%), and the difference is statistically significant at 5% confidence level. Grossman et al (2001) support these findings using a dataset of defaulted loans and bonds from 1997-2000.

Controversially, Acharya, Bharath and Srinivasan (2006) find that when industry effects are controlled for, macroeconomic effects are no longer significant. Indeed, industry conditions at the time of default are found to be robust and important determinants of recovery rates. They show that creditors of defaulted firms recover significantly lower amounts in present-value terms when the industry of defaulted firms is in distress and also when non-defaulted firms are rather illiquid and if their debt is collateralized by specific assets that are not easily re-deployable into other sectors. Also, they find that there is little effect of macroeconomic conditions over and above the industry conditions and the latter is robust even with the inclusion of macroeconomic factors. The authors suggest that the linkage between bond market aggregate variables and recoveries arises due to supply-side effects in segmented bond markets**,** and that this may be a manifestation of Shleifer and Vishny**'**s (1992) industry equilibrium effect. That is, macroeconomic variables and bond market conditions may be picking up the effect of omitted industry conditions. The importance of the "industry" factor in determining *LGD* is highlighted by Schuermann (2005).

It is also observed that recoveries are systematically and substantially lower in recessions, meaning that an *LGD* model should incorporate at least some macro/business-cycle variables. Frye (2000) analyses Moody's data to conclude that in a recession, the recoveries can be about a third lower than in expansion. Carey (1998) demonstrates that the impact of business cycle is much more substantial for sub-investment grade borrowers, whereas for investment grade borrowers the difference is modest.

We can thus summarize that a fully-specified *LGD* model should take into account debt seniority, presence and quality of collateral, industry and, possibly, timing of business cycle. In application to deposit insurance, seniority can be ignored provided that all claims of the deposit insurer have the same level of seniority. Also, either macroeconomic or financial sector variables can be used to control for the state of the environment. Finally, we can suggest that although deposit insurer's claims are not collateralized, the composition of the defaulted lending institution's assets should have a major impact on the recoveries received by the deposit insurer.

2.2.3 Commercial applications

In the existing commercial models of credit risk, the recovery rate is typically taken as an exogenous constant parameter or a stochastic variable independent from *PD*.

CreditMetrics (Gupton, Finger and Bhatia, 1997) assumes that the recovery rates are distributed according to a beta distribution with the mean and standard deviation being the historical averages for the respective seniority class calculated by an independent rating agency. As it has been previously mentioned, empirical *LGD* distributions are generally bimodal. Thus, beta distribution is not the best possible approximation. More importantly, the use of backward-looking historical averages does not reflect the current stage of the business cycle, meaning that an upward bias exists when the economy is growing, and a downward bias exists during a recession.

Similarly, CreditRisk+ (CSFB, 1997) and CreditPortfolioView (Wilson, 1997) require recovery rates as exogenous inputs, implying that *LGD* modeling should be based on rating agencies' historical averages. KMV's CreditPortfolioManager also relies on user input of exogenous *LGD* values, however, an additional statistical model, LossCalc, has been developed to estimate recovery rates with more precision (Gupton and Stein, 2002).

In contrast with traditional static models, LossCalc is dynamic and able to give a more exact specification of *LGD* horizon that incorporates cyclic and firm specific effects. LossCalc incorporates several groups of factors:

- **debt-type:** (i.e., loan, bond, and preferred stock) and **seniority grade** (e.g., secured, senior unsecured, subordinate, etc.);
- **firm specific capital structure**: leverage and seniority standing;
- **industry:** moving average of industry recoveries; banking industry indicator[12];
- **macroeconomic:** one-year median RiscCalc default probability; Moody's Bankrupt Bond Index; trailing 12-month speculative grade default rate; changes in the index of Leading Economic Indicators[13].

LossCalc defines recovery on a defaulted instrument (loan, bond or preferred stock) as its market value (bid-side market quote) approximately one-month after default. The model assumes that *LGD* follows a beta distribution and then runs a normal transformation to estimate the *LGD* using a least squares regression on non-parametric transforms of the initial variables. Conditional confidence intervals are then estimated to derive probabilistic estimates of recovery rates. The resultant confidence intervals are demonstrated to be tighter than those derived from historical estimates of mean and standard deviation of *LGD* (Gupton and Stein, 2002).

LossCalc approach appears to be superior to the approaches suggested by other industrial models: it accounts for the seniority of debt, as well as for the current stage of the business cycle and the industry trends. Additionally, it allows for correlation with the default probabilities by explicitly incorporating the one-year median RiskCalc default probability and trailing 12-month speculative grade default rate as macroeconomic explanatory variables. However, as any econometric model it relies on abundance and relevance of historic data.

[12] Gupton and Stein (2002) advocate the use of banking industry indicator by their empiric finding that recoveries for bank defaults are consistently low across time.

[13] composite indicators of the US economy provided by a commercial vendor.

3 Interrelation of inputs

3.1 Default correlation

3.1.1 Theoretical framework

3.1.1.1 Factor models

Due to their simplicity, factor models have received a wide recognition in off-the-shelf credit risk models. The calibration of factor models is usually carried out by logit or probit regression, depending on the assumptions about the distribution of the factors. Schönbucher (2000), Finger (1999), and Frey, McNeil & Nyfeler (2001) present a detailed illustration of these models.

3.1.1.2 Structural models

The most natural way to introduce default dependences between firms in structural models is by correlating the firms' asset processes. Describing a firm's asset value process by geometrical Brownian motion means that defaults are perfectly predictable. This approach is used in Vasicek formula, also called the asymptotic single risk factor approach, which forms the heart of the IRB[14] formula of Basel II. Vasicek (1987) assumes the portfolio is comprised of similar borrowers with the same *PD*s. Given correlation between returns on the assets of the borrowers in the portfolio and given the level of confidence, his formula specifies the level of capital (expressed as a percent of *EAD*) that is required to prevent the bank from going bankrupt in one year, assuming no recovery (i.e., *LGD*=1). Vasicek (1991) also provides an asymptotic

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[14] Internal Rating Based approach, proposed by Basle Committee, permits the lending institutions to use their own internal measures for key drivers of credit risk as primary inputs to the capital calculation, subject to meeting certain conditions and to explicit supervisory approval. All institutions using the IRB approach will be allowed to determine the borrowers' probabilities of default while those using the advanced IRB approach will also be permitted to rely on own estimates of loss given default and exposure at default on an exposure-by-exposure basis. These risk measures are converted into risk weights and regulatory capital requirements by means of risk weight formulas specified by the Basel Committee

analytical expression for cumulative distribution function of the percentage of loss on a large portfolio, assuming that the bank's loan portfolio is infinitely fine grained. Tarashev and Zhu (2007) observe that despite Vasicek's (1987 and 1991) assumptions typically do not hold true in practice, violation of these assumptions does not result in material biases in the resultant loss distribution function. In contrast, even minimal errors in default correlation and/or asset return distributions tails' estimates result in substantially biased estimates of portfolio credit risk. Adequate estimation of default correlations is thus one of the cornerstones of an efficient credit risk model.

Huang et al (2006) suggest a higher order saddlepoint approximation for estimation of asset returns correlation in a concentrated credit portfolio. This technique allows for a more adequate estimate of obligors' returns distributions, and thus, for an enhanced estimate of portfolio credit risk. Huang et al (2006) also demonstrate that this approach can be extended for a more general multi-factor returns correlation model with stochastic loss given default. Given the efficiency and feasibility of this approach, it can be recommended that it is adopted by the more advanced risk insurers that are interested in more efficient default correlation and unexpected loss estimates for highly concentrated portfolios of contingent liabilities. Tasche and Theiler (2004) observe that Basel II approach is inadequate for assessing the economic cpital requirements of a bank with concentrated credit portfolio. They suggest an alternative semi-parametric approach based on a Conditional Value-at-Risk (CVaR), arguing that this risk indicator that is more sensitive to portfolio concentration. Kupiec (2008) suggests a generalization of the Vasicek (1987 and 1991) model that allows for analytical derivation of credit portfolio loss function distribution under the assumption that exposure at default and loss given default of each obligor are correlated stochastic processes.

Vasicek (1987 and 1991) model and its derivatives are often criticized on the grounds that these models accommodate for predictable defaults. One way of getting rid of the default predictability would be to introduce jump components in the firms' asset processes. Those jump components could be either correlated or uncorrelated across firms. Correlated jump components, besides making defaults unpredictable, would also account for credit risk contagion effects. The main problem lies in the calibration of those jump components. Giesecke (2004b) proposes a model in which the default thresholds are constant and known, and in which the distribution of the historical lows of firms' asset value processes are linked through a copula[15] function.

Cyclical default correlation does not account for all the credit risk dependence between firms. Giesecke (2004a & 2005) extends structural models for default correlation to incorporate credit risk contagion effects under incomplete information. The default of one firm can trigger the default of other related firms. Furthermore, default times tend to concentrate in some periods of time in which the probability of default of all firms is increased and which can not be totally, or even partially, explained by the firms' common dependence on some macroeconomic factors. Contagion effects can arise in this setting by direct links between firms in terms of, for example, commercial or financial relationships. The news about the default of one firm have a big impact in the credit quality of other related firms which is immediately reflected in their default probabilities.

In structural first passage models we assume that investors have complete information about both asset processes and default thresholds, so they always know the nearness of default for each firm, i.e. the distance between the actual level of the firm's assets and its default threshold. Giesecke (2004) introduces contagion effects in the model by relaxing the assumption that investors have complete information about the default thresholds of the firms, while maintaining the assumption of complete information about the diffusion process governing the dynamics of the firms' asset process. The incomplete information about the level of the default thresholds and the fact that those levels are dependent between firms represent the source of credit risk contagion. Investors form a belief, in terms of both individual and joint distribution functions, about the level of the firms' default thresholds. Each time one of the firms default, the true level of its default threshold is revealed, and investors use this new information to update

[15] Copula is one of the possible ways to describe a dependence of random variables. Formally, copula is the joint multivariate distribution function for a multivariate distribution with standard uniform marginal distributions. Any joint multivariate distribution function can be expressed as a copula with arguments equal to univariate marginal distribution functions of respective variables.

their beliefs about the default thresholds of the rest of the firms. This sudden update of the investors' perceptions about the default thresholds of the firm, and thus about the nearness of default for each firm, introduces the default contagion effects in the models.

This model allows the introduction of default correlation both through dependences between firms' asset values, cyclical default correlations, and through dependences between firms' default barriers, i.e. contagion effect. The major problem of this approach is to calibrate and estimate the default threshold copula. Giesecke (2005) provides a further discussion on choosing and calibrating the copula.

The uncertainty of the default point is further discussed by Galai, Raviv and Wiener (2005). The authors argue that in practice default does not necessarily lead to immediate change of control or to liquidation of the firm's assets by its debtholders. To consider the impact of this on the valuation of corporate securities, they develop a model in which liquidation is driven by a state variable that accumulates with time and severity of distress. The suggested model can be viewed as a generalization of the Merton's framework, in which liquidation occurs only upon debt maturity, and the Black-Cox model, in which reorganization of the firm's assets is invoked when a minimum threshold is violated during the lifetime of the debt.

3.1.1.3 Reduced form models

Recent literature elaborates on three approaches to incorporating default dependencies within the framework of reduced form models. The first approach introduces correlation in the firms' default intensities, making them dependent on a set of common variables and on a firm specific factor. These models have received the name of conditionally independent defaults (CID) models, because as they are conditioned to the realization of the state variables, the firm's default intensities are independent as are the default times that they generate. Apparently, the main drawback of these models is that they do not generate sufficiently high default correlations. However, Yu (2002a) indicates that this is not a problem of the models per se, but rather an indication of the lack of sophistication in the choice of the state variables.

Two direct extensions of the CID approach try to introduce more default correlation in the models. One is the possibility of joint jumps in the default intensities (Duffie and Singleton, 1999b) and the other is the possibility of default-event triggers that cause joint defaults (Duffie and Singleton, 1999b; Kijima, 2000; and Kijima and Muromachi, 2000). An accessible formal outline of conditional independent approach to modeling default correlations is provided in Elizade (2003).

Duffie and Singleton (1999b) propose two ways out of the low correlation problem. First, they introduce correlation to the firm's jump processes, keeping unchanged the characteristics of the individual intensities. They postulate that each firm's jump component consists of two kinds of jumps, joint jumps and idiosyncratic jumps. The joint jump process has Poisson intensity and an exponentially distributed size. The idiosyncratic jump (independent across firms) is set to have an exponentially distributed size and intensity. The second alternative considers the possibility of simultaneous defaults triggered by common credit events, at which several obligors can default with positive probability. If given the occurrence of a common shock, the firm's default probability is less than one. This common shock is called nonfatal shock, whereas if this probability is one, the common shock is called fatal shock. In addition to the common credit events, each entity can experience default through an idiosyncratic Poisson process, which is independent across firms. Duffie and Singleton (1999b) also propose algorithms to simulate default times within these two frameworks.

The criticisms that the joint credit event approach has received stem from the fact that it is unrealistic that several firms default at exactly the same time, and also from the fact that after a common credit event that makes some obligors default, the intensity of other related obligors that do not default does not change at all.

The literature on credit risk correlation has criticized the CID approach, arguing that it generates low levels of default correlation when compared with empirical default correlations. However, Yu (2005) argues that the default correlation in reduced-form models can be quite sensitive to the common factor structure imposed on individual default intensities. The author supports his argument using numerical examples calibrated to two studies, namely Duffee (1999), where there are two common factors, both of which extracted from Treasury yields, and Driessen (2005), where two additional common factors capture the co-movement of corporate credit spreads. The first case implies a default correlation much lower than empirical observations, while the second case implies comparable or even higher values. Therefore, the alleged low default correlation in reduced-form models may have more to do with an inadequate common factor structure than the assumption of conditional independence.

Contagion models take CID models one step further, introducing to the model two empirical facts: that the default of one firm can trigger the default of other related firms and that default times tend to concentrate in certain periods of time, in which the default probability of all firms is increased. Joint credit events model summarized previously differs from contagion mechanisms in that if an obligor does not experience a default, its intensity does not change due to the default of any related obligor. The literature of default contagion includes two approaches: the infectious defaults model of Davis and Lo (2001), and the propensity model proposed by Jarrow and Yu (2001). The main issues to be resolved concerning these two models are associated with difficulties in their calibration to market prices.

In order to account for the clustering of default in specific periods, Jarrow and Yu (2001) extend CID models to account for counterparty risk, i.e. the risk that the default of a firm may increase the default probability of other firms with which it has commercial or financial relationships. This allows them to introduce extra-default dependence in CID models to account for default clustering. In a first attempt, Jarrow and Yu (2001) assume that the default intensity of a firm depends on the status (default/not default) of the rest of the firms, i.e. symmetric dependence. However, symmetric dependence introduces circularity in the model, which they refer to as looping defaults, which makes it extremely difficult and troublesome to construct and derive the joint distribution of default times. Jarrow and Yu (2001) then restrict the structure of the model to avoid the problem of looping defaults. They differentiate between primary firms and secondary firms. First, they derive the default intensity of primary firms, using a CID model. The primary firm intensities do not depend on the default status of any other firm. If a primary firm defaults, this increases the default intensities of secondary firms, but not the opposite. This model introduces a new source of default correlation between secondary firms, and also between primary and secondary firms, but it does not solve the drawbacks of low correlation between primary firms, which CID models apparently imply. Yu (2002a) and Frey and Backhaus (2003) offer a further extension of Jarrow and Yu (2001) model.

Davis and Lo (2001) suggest a model of infectious defaults that is available in static version that only considers the number of defaults in a given time period, and a dynamic version in which the timing of default is also incorporated. In the dynamic version of the model, each firm has an initial hazard rate which can be constant, time dependent or follow a CID model. When a default occurs, the default intensity of all remaining firms is increased by an enhancement factor greater than 1. This augmented intensity remains for an exponentially distributed period of time, after which the enhancement factor disappears. During the period of augmented intensity, the default probabilities of all firms increase, reflecting the risk of default contagion.

In correlated intensity models and contagion models the specification of the individual intensities includes all the default dependence structure between firms. In contrast, the copula approach separates individual default probabilities from the credit risk dependence structure. The copula function takes as inputs the marginal probabilities and introduces the dependence structure to generate joint probabilities. The first approach was introduced by Li (1999) and represents one of the first attempts to use copula theory systematically in credit risk modeling. Li's approach takes as inputs the marginal default (survival) probabilities of each firm and derives the joint probabilities using a copula function. This can be regarded as a generalization of the CreditMetrics approach. The second approach was introduced by Schönbucher and Schubert

(2001). Their idea was to link the random default thresholds with a copula. In contrast to the models of Jarrow and Yu (2001) and Davis and Lo (2001), this approach allows the contagion effects to arise endogenously through the use of the copula. This concept is further developed by Rogge and Schönbucher (2003).

3.1.1.4 Merging structural and reduced form models

A recent stream of research suggests merging structural and reduced-form models to provide a more precise estimation of the default correlation structure. For instance, Madan and Unal (2000) use the reduced form framework and model the hazard rate as a linear function depending on the level of the interest rates and the logarithm of the value of the firm's assets; hence the parameters of the hazard rate process can be linked to firm-specific information. However, this specification of the hazard rate of default can admit negative values with positive probabilities and for very low values of the assets of the firm default could still not occur.

Zhou (1997) models the firm's assets value process by including a jump component. He derives an approximate solution and obtains analytical solutions for defaultable bonds in the context of a simplified model where default occurs at some predetermined dates. Cathcart and El-Jahel (2002) allow for expected and unexpected default by introducing a stochastic hazard rate of default that admits a lower boundary at which default becomes a certain event. Duffie and Lando (1997) derive a hazard rate of default based on an unobservable value of the firm. In their setup, imperfect accounting data introduces an uncertainty about the current level of the assets of the firm relative to the default boundary which allows for positive credit spreads at short maturities.

Cathcart and El-Jahel (2002) suggest a closed form solution for the pricing of defaultable bonds and default correlation based on an alternative definition of default. In their specification, it occurs when the value of the assets of the firm hits a stochastic boundary of default or according to a jump-event. The firm will default if its assets drop below the discounted face value of debt or if a sudden loss strikes unexpectedly. The hazard rate captures the probability of extraordinary loss. It is function of a stochastic variable that is assumed to follow a mean reverting square root process[16]. The square-root process assumption precludes the possibility of negative hazard rates. This specification for the occurrence of default encompasses realistic default scenarios and is solved in closed form. The short-term credit spreads generated within the proposed framework are consistent with empirical findings. Finally, it allows deriving default correlations and joint default probability between firms in closed form.

Documenting that systematic time-variation in default risk is driven more by an economy-wide volatility factor than by changing debt levels, Das et al (2004) propose a reduced-form stochastic framework to model joint default risk with these properties. The calibrated system shows that there are substantial differences in default distributions across economic regimes.

3.1.2 Commercial applications

To estimate the joint credit rating migration probabilities, CreditMetrics relies on equity prices for publicly traded companies as proxies to calculate asset correlations. Multi-factor analysis is then used to reduce the dimensionality of the correlation matrix. This approach implies that the asset value of the firm can be decomposed into a number of common industry and country factors, and also an uncorrelated idiosyncratic component. Country and industry weights are regarded as external inputs.

Similar to CreditMetrics, KMV model derives asset return correlations from a structural model which links correlations to fundamental factors. KMV constructs a three-layer factor structure model. On the first level, systematic risk is captured by a single composite index, which is firm specific. This index is constructed on the second level as a weighted sum of the firm's exposure to country and industry factors. At the third level of the factor structure the risk of countries and industries is further decomposed into systematic and idiosyncratic components. The systematic component is captured by basic factors like: global economic effect, regional

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[16] This process is a solution of stochastic differential equation of special form; it was introduced in finance theory by Cox Ingersoll and Ross (1985) to describe short interest rate evolution with mean reversion property and ensuring positivity of solution.

factor effect and sector factor effect. While the common factor is firm-specific, the third level factors are the same for all countries and all industries.

CreditRisk+ does not make any implications regarding default correlations. It is further suggested that the average number of defaults, i.e. the sum of default probabilities over the portfolio, is a random value, without any further implications regarding the common risk drivers behind the probabilities of default of individual obligors.

CreditPortfolioView suggests that all default probabilities are dependant on the same set of macroeconomic and industrial variables with different sensitivities. Thus, the resultant default probabilities are inherently correlated with each other, allowing for analytical derivation of the correlation matrix. The cyclical effects are further incorporated into the model by adjusting the unconditional transition matrix for the business cycle coefficient. The coefficient is estimated as the ratio of simulated default probability for a speculative grade obligor during the current stage of the business cycle to the historical average default probability for the same obligor.

It can be summarized that off-the-shelf models estimate default correlations relying on either the equity price data or industrial and macroeconomic variables. In case of deposit insurers, all insured institutions fall within the same industry and market data is hardly ever available and sufficient to produce robust estimates. Thus, a simplified approach to estimating default correlations can be implemented.

Bennett (2002) discusses the model developed by Oliver, Wyman and Company for explicit modeling of the US FDIC's loss distribution. The default correlation structure proposed by the model assumes that all elements in the portfolio can be grouped into several homogenous buckets defined by the user. The stochastic properties of the default of institutions within each bucket are assumed to be the same. Bennett (2002) performs a sensitivity test of the model with under various bucketing assumptions: the institutions are put into 25 buckets according to their size and region, CAMELS rating and region, CAMELS rating and size, or industrial specialization and region. Different bucketing schemes result in different implied solvency levels of the FDIC.

Smirnov et al (2006) perform an advanced clustering analysis of the Russian inter-bank lending market to discover stable lending patterns within the industry that can have a major impact on the default correlation of the insured institutions. Under a simplified approach, it is assumed that all banks active in the interbank lending are characterized with the same level of default correlation, whereas the defaults non-participant banks are not correlated. Historical data is then used to calibrate the estimated correlation. It is also demonstrated that even minimal correlation level has a substantial impact on the DIF requirements.

3.2 Impact of business cycle

In the previous section, we reviewed the literature concerning the correlation structure of default probabilities to find that the correlation can partially be explained by the reliance upon the state variables representing the stage of the business cycle. In this section, we review the models that provide explicit linkage between *PD* and *LGD*, on the one hand, and the business cycle, on the other.

3.2.1 Cyclical effects on probability of default

3.2.1.1 Structural models

Structural models measure the cyclical impact on *PD* by incorporating systematic risk factors into the specification of the stochastic asset diffusion process. Since default occurs in a structural model when the market value of assets falls to the default point (set equal to the face value of debt), then the *PD* depends on the distance between the market value of assets and the default point during the credit horizon period. Forecasting the forward distribution of asset values is therefore critical to the determination of *PD* in a structural model. The consensus in this branch of the literature is that asset values and *PD*s tend to be positively correlated across obligors. Moreover, *PD* is time-varying and regime dependent.

Fridson, Garman and Wu (1997) find a relation between macroeconomic conditions and *PD*. In particular, they find that as real interest rates increase, asset values decrease, thereby increasing the estimate of *PD* in a structural model. They find a two year lag in the interest rate effect because of the existence of a cushion of cash reserves or a lag until debt payment date that may allow even insolvent firms to delay default. Barnhill and Maxwell (2002) report a correlation coefficient of –0.33 between the risk-free interest rates and the S&P500 market index, meaning that the Fridson, Garman and Wu (1997) result implies a positive correlation between *PD* and the overall market index. Barnhill and Maxwell (2002) also simulate asset distributions that are conditional on macroeconomic conditions. They find that systematic risk exposure increases as credit quality deteriorates.

Gersbach and Lipponer (2000) also find that default correlations increase (decrease) as credit quality deteriorates (improves). The authors also examine the impact of macroeconomic shocks (measured as interest rate shocks) on default correlations for loan portfolios, holding constant both asset correlations and default probabilities. They find that macroeconomic shocks increase positive default correlations, thereby engendering procyclical effects as portfolio diversification benefits decline (i.e., both *PD* and default correlations increase) in economic downturns. This procyclical effect is significant – on the order of 30% of the increase in credit risk when initial *PD* is 5% for initial default correlations of 14.6%.

This result is supported by a paper by Collin-Dufresne and Goldstein (2001) that focuses on the relationship between the market value of assets and the default point. Thus, as the default risk-free rate increases, asset values decline, thereby causing an increase in *PD*s, or a positive correlation between changes in default risk-free interest rates and default risk.

Zhou (2001) uses a first passage time model to ascertain the time until the asset value reaches the default point, which is assumed to be fixed at the value of short term liabilities plus one half of all long term liabilities. His results are consistent with those of the previous studies in that he finds stronger macroeconomic effects for low credit quality firms than for high credit quality firms. Since the credit quality of the firm is itself dynamic, Zhou (2001) contends that the cyclical effects on *PD* are also dynamic. Using an assumption that the correlation between asset values is 40%, Zhou (2001) finds that default correlations increase as the time to maturity increases and as the credit quality decreases. However, Zhou (2001) also finds evidence that default (particularly for short maturity debt) is idiosyncratic and related to unexplained jumps in the asset diffusion process.

Crouhy, Galai and Mark (2000, 2001) also find that the most speculative risk classifications' default probabilities are most sensitive to shifts in macroeconomic conditions. That is, *PD* correlations are highest for low quality firms. In particular, they find the existence of an asymmetric procyclical impact on *PD*s such that default probabilities increase significantly during economic downturns, but do not decrease significantly during economic upturns. That is, a recession is sufficient to force many marginal firms into default, thereby causing large increases in both *PD*s and default correlations for these firms. In contrast, an economic boom is insufficient to lift many of these firms' credit quality, thereby reducing the correlation across firm *PD*s. Stated simply, business recovery is driven more by firm-specific factors, whereas business failure is more systematic.

Longin and Solnik (2001) also find evidence of asymmetric procyclicality. Using extreme value theory, they find increases in correlations across international equity markets during bear markets, but not in bull markets. Since structural models use equity prices to estimate *PD*, Longin and Solnik's (2001) results imply that default correlations should increase during economic downturns, but not necessarily during economic upturns.

Erlenmaier and Gersbach (2001) may resolve some of the controversy about whether default correlations are directly or inversely related to *PD*. Using a structural model and a fixed, exogenous *LGD*, they divide the correlation effect into a skewness effect and a distance-of default effect. Systematic risk factors that increase *PD* levels tend to move the observations into the extreme portions of the default distribution that are more highly skewed, meaning that there is more divergence among the *PD*s for individual firms. Since the greater the skewness, the less information is revealed about the correlated underlying asset returns, then increases in skewness result in decreases in default correlations. Thus, the relationship between default correlations and the *PD* resembles a negative quadratic function: it increases for the region up until *PD* = 50% and then decreases thereafter. However, there is a countervailing distance-of default effect, which is monotonically decreasing as *PD* increases. That is, if one firm's *PD* increases and the other firm's *PD* stays the same, it is tautological that both firms' *PD*s will diverge and the correlation between their *PD*s will decrease. The observed relationship between the level of *PD* and the default correlation nets the skewness effect and the offsetting distance-to-default effect. Based on their simulation results, Erlenmaier and Gersbach (2001) contend that the skewness effect dominates the distance-to-default effect in the relevant range. Therefore, default correlations tend to increase as *PD* increases. They also observe that the impact of cyclical effects on *PD* levels and correlations is only part of the picture, and find that the standard deviation of default rates varies throughout the business cycle. Extreme economic conditions (booms and busts) are characterized by two and three fold increases in portfolio standard deviation in addition to shifts in default correlations.

Allen and Saunders (2003) provide a survey of cyclical effects in credit risk measurement models and also argue that in addition to the systematic fluctuations in firms' asset values, the existence of procyclical shifts in the default point (i.e., leverage amounts) may also induce cyclical *PD*s. When economic conditions deteriorate, shareholders may be more likely to extract concessions from debtholders that lower the default point. Possible deviations from absolute priority occur when liquidation costs are very high. Thus, debtholders may be willing to reduce the face value of the debt so as to avoid the high costs of bankruptcy and liquidation of assets. If lenders are more likely to renegotiate debt in recessions than during expansions, and if asset volatilities are unchanged, then decreases (increases) in economic activity would coincide with decreases (increases) in *PD*. However, if asset volatility increases (decreases) during recessions (expansions), there would be a procyclical pattern in *PD* such that *PD* increases during recessions and decreases during expansions. Allen and Saunders (2003) conclude that which of these two effects dominates is a matter for empirical investigation, although the anecdotal evidence suggests that the asset volatility effect dominates the effect of a cyclical shift in the default point.

3.2.1.2 Reduced form models

Reduced form models decompose observed credit spreads to detect the term structure of default probabilities. Thus, reduced form models do not examine the structural factors leading to default. *PD* is instead modeled using the stochastic intensity function that best fits the yield curve data. If credit spread is a pure risk premium for credit risk exposure, then risk neutral security pricing implies that credit spread equals *PD* x *LGD*. In this section, we discuss how reduced form models decompose the credit spread in order to solve for *PD*.

Duffie and Singleton (1998) model an intensity function with both idiosyncratic and systematic factors. The model can incorporate multiple systematic factors. The cyclical effect is observed in the correlated Poisson arrivals of randomly sized jumps in default intensities. Moreover, Duffie and Singleton (1999) and Lando (1998) model the cyclical component as a function of the short term risk-free interest rate (where interest rates are inversely correlated with the market index). However, this specification does not obtain estimates of *PD* that exhibit the cyclicality in *PD*s observed in anecdotal evidence.

Geyer, Kossmeier and Pichler (2001) apply the Duffie and Singleton (1999) model to European government bond spreads, defined to be the spread over German sovereign bonds (assumed to be default risk-free). They find strong evidence of a global systematic risk factor as well as idiosyncratic country risk factors for each issuer over the period 1999-2000. The global risk factor represents the average level of yield spreads across all countries and across all maturities.

Das, Freed, Geng and Kapadia (2001) and Das, Fong and Geng (2001) use an intensity-based model to detect cyclical default probabilities. Their results parallel those of Crouhy, Galai and Mark (2000, 2001) in that *PD* correlations among US non-financial public firms over the period January 1987 to October 2000 are estimated to be higher when markets move down (ie, *PD* levels are high on average) in contrast to when markets move up (*PD* levels are low). Their results contradict those discussed previously, e.g. Barnhill and Maxwell (2002), Gersbach and Lipponer (2000), Erlenmaier and Gersbach (2001), Crouhy et al (2000, 2001) and Zhou (2001) in that default correlations increase as credit quality improves. That is, *PD* across high credit quality firms may be higher at times than for low credit quality firms because high quality firms have less idiosyncratic risk in their balance sheets than do low quality firms. Moreover, Das, Freed, Geng and Kapadia (2001) hypothesize that *PD* correlations fluctuate over time. They use US bond data over the period 1987-2000 to estimate a switching of regression regimes model that endogenizes the time period cut-off points. The time period regimes do not conform to business cycles, suggesting that fluctuations in *PD* correlations are not necessarily cyclical. Moreover, the highest correlation is found for the earliest period in their sample: January 1987 –

April 1990, a period that includes both recession and non-recession years. Das, Fong and Geng (2001) show that ignoring these time-varying correlations in default probabilities results in substantial underestimates of credit risk exposure.

Jarrow and Yu (2001) consider a doubly stochastic Poisson process[17]. The default intensity depends on macroeconomic factors and an interdependence term linking firms across industries and sectors. Thus, correlations across *PD*s arise because of both a systematic risk factor and a counterparty risk factor that is essentially an exposure to other firms' idiosyncratic risk. This counterparty risk may emanate from exposure to suppliers as in vertically integrated manufacturing processes, access to capital, and contagion effects. The researchers find that consideration of counterparty risk factors results in estimates of *PD* that exhibit the observed clustering in defaults found during economic downturns.

Bakshi, Madan and Zhang (2001) estimate a three factor credit risk model that depends on systematic (observable economic) factors and firm-specific distress variables (such as leverage, book-to-market, profitability, lagged credit spread, and scaled equity price). The systematic factors are the default risk-free interest rate and its stochastic long run mean. They find that the interest rate factors are important determinants of the credit spread. Moreover, the idiosyncratic factors representing firm distress (particularly the leverage and book-to-market variables) reduce out-of-sample fitting errors for a sample of US corporate bonds (without embedded options) issued from January 1973 to March 1998. However, the model performs better for high credit quality bonds than for higher risk bonds.

3.2.2 Cyclical effects on loss given default

Virtually no research has investigated the impact of structural factors into *LGD* procyclicality. For example, bankruptcy rules differ across countries and across time periods. During periods of economic crisis, bankruptcy rules are often leniently applied, as in Japan during the past decade. Moreover, as lenders prove more amenable to renegotiation during

[17] Under doubly stochastic Poisson process, or the Cox process, the claim intensity function is assumed to be stochastic. That is, this process is a Poisson process conditional on the fact that realization of this random intensity function is known

recessions, *PD* may decrease (since insolvent firms are allowed forbearance in order to avoid default), but recovery rates also may decrease. This results in procyclical increases in *LGD* during bad economic times.

Allen and Saunders (2003) point out that the stringency of bankruptcy rules differs dramatically across countries. In the US, management is granted an exclusivity period immediately upon entering Chapter 11 during which the management cannot be removed (unless the courts find evidence of fraudulent behavior). During this period (which may last as long as nine months), the managers have a choice – they can either undertake activities to increase firm value or they can pursue their own self-interest and allow firm value to deteriorate further. To the extent that management concern about future employment prospects and personal reputation, as well as short term consumption of perquisites, outweighs the manager's long term interest in the distressed firm, the end of the exclusivity period may find the firm's creditors with substantially impaired assets, thereby reducing recovery rates and increasing *LGD*. To the extent that procyclicality affects the likelihood of bankruptcy, then the legal and regulatory environment governing bankruptcy administration is relevant for credit risk assessment. To our knowledge, this has not been incorporated into either academic or proprietary models.

Most academic and proprietary models make the simplifying assumption that recovery rates are exogenously determined. Indeed, the earliest models assumed an *LGD* that was a fixed, known fraction of the debt value. This was a critical assumption for reduced form models that enabled them to disentangle the *PD* from the *LGD* in the observed credit spread. Second generation credit risk measurement models have just begun to address the cyclicality in *LGD*.

3.2.2.1 Structural models

Structural models evaluate the *PD* as the likelihood that the market value of assets will fall to the default point (the debt value). Once default occurs, debtholders receive the market value of the firm's assets. Thus, if there is a cyclical component built into asset valuations, and then it also impacts recovery rates. Despite this, such structural models, as Kim, Ramaswamy and Sundaresan (1993), Hull and White (1995), and Longstaff and Schwartz (1995) assume that *LGD* is exogenously determined. An exception to this is the papers by Frye (2000a and 2000b). Frye (2000b) uses a bond database to find evidence of cyclical recovery rates. Collateral values fluctuate with economic conditions; with recovery rates sometimes declining 20-25% in severe economic downturns. Collateral values are particularly sensitive to economic downturns for three reasons: (1) the direct effect of systematic risk exposure; (2) an indirect effect if distressed obligors cut back on asset/collateral maintenance and control; and (3) an indirect effect if distressed lenders dump assets/collateral in fire sale liquidations.

Frye (2000a) models collateral values as a function of both idiosyncratic and systematic risk factors, finding a considerable impact of cyclical factors on expected losses. Frye (2000b) estimates that the correlation between asset values and the systematic risk factor (for a US bond database over the period 1983-1997) is 23% and that the correlation between collateral values and the systematic risk factor is almost the same: 17%.

Erlenmaier and Gersbach (2001) consider endogenous recovery rates that are a fixed fraction of asset values. The impact of endogenous *LGD* is to increase default correlations as compared to the exogenous case. Moreover, the relationship between *PD* levels and default correlations is exacerbated when *LGD* is endogenously determined by asset values. However, this result assumes that the cyclical effect is constant over time. If instead there are regime shifts that affect the firm's exposure to systematic and idiosyncratic risk factors, then the default correlation function will shift over time. Indeed, extreme outcomes may result in greater default correlations because information is revealed about the underlying regime state. Thus, if *PD* and *LGD* both increase in economic downturns and decrease in economic upturns, then the cyclical effect (as measured by both default correlations and *LGD* correlations) will be more pronounced.

3.2.2.2 Reduced form models

Reduced form models estimate the default intensity function using observed credit spreads on risky debt. The credit spread is defined as *PD* x *LGD*. Thus, reduced form models must find some way to disentangle the *PD* from the *LGD* in each observation of the credit spread. Many of the earlier reduced form models focused on modeling the default intensity, *PD*, in order to disentangle these two components of the credit spread. Their simplifying assumptions that the *LGD* was either constant or proportional to bond value were counterfactual. However, observed recovery rates are volatile and appear to have a cyclical component. Moreover, the default intensity also fluctuates with the business cycle and systematic risk conditions.

Das and Tufano (1996) allow a proportional *LGD* to vary over time, but maintain the assumption of independence between *LGD* and *PD*. Duffie and Singleton (1999) allow for (economic) state dependence of both *LGD* and *PD*, as well as interdependence between *LGD* and *PD*; however, they assume independence between firm asset value and the *LGD* and *PD* processes, an assumption that does not hold if, for example, the debt obligation is a large part of the issuer's capital structure. The pure recovery model of Unal, Madan and Guntay (2001) decomposes the difference between the prices of senior versus junior debt in order to obtain a measure of recovery rates on senior debt relative to junior debt that is independent of default probabilities. The recovery rate is conditioned on the business cycle (measured using macroeconomic factors) and firm-specific information.

Miu and Ozdemir (2005) further elaborate on the downturn *LGD* concept suggested by the BIS (2005). The authors argue that the conservatism in point in time *LGD* estimates can account for the lack of correlations. They also demonstrate that use of bottom-of-the-cycle *LGD* estimate is equivalent to increasing the risk horizon. Examining the correlations between *PD* and *LGD* in detail, the researchers show that there are different types of correlations in question: both *PD*s and *LGD*s can be characterized by pair-wise and cross-correlation. Using historical default data of a middle-market loan portfolio, Miu and Ozdemir (2005) estimate the correlations of *LGD* risk drivers among different obligors and the correlation of systematic *PD* and *LGD* risk factors. The former is found to be small whereas the latter can be substantial. The analysis provided illustrates that even at a moderate level of idiosyncratic and systematic correlation, the mean *LGD* needs to be increased substantially in order to achieve the correct economic capital under a model where correlations are ignored. However, the exact markup depends on the seniority and collateralization of the loan.

3.2.3 Cyclical effects on relation between default probability and loss given default

Over the last few years, new approaches explicitly modeling and empirically investigating the relationship between *PD* and *LGD* have been developed. These models include Bakshi et al (2001); Jokivuolle and Peura (2003); Frye (2000); Jarrow (2001); Hu and Perraudin (2002); and Carey and Gordy (2003); Altman, Brady, Resti and Sironi (2003, 2005); and Acharya, Bharath and Srinivasan (2003, 2007).

Bakshi et al (2001) offer an enhanced reduced-form model that allows for a flexible correlation between the risk-free rate, the default probability and the recovery rate. Based on some evidence published by rating agencies, they force recovery rates to be negatively associated with default probability. They find some strong support for this hypothesis through the analysis of a sample of BBB-rated corporate bonds: more precisely, their empirical results show that, on average, a 4% worsening in the (risk-neutral) hazard rate is associated with a 1% decline in (risk-neutral) recovery rates.

A rather different approach is the one proposed by Jokivuolle and Peura (2003). The authors present a model for bank loans in which collateral value is correlated with the *PD*. They use the option pricing framework for modeling risky debt: the borrowing firm's total asset value triggers the event of default. However, the firm's asset value does not determine the *RR*. Rather, the collateral value is in turn assumed to be the only stochastic element determining recovery. Because of this assumption, the model can be implemented using an exogenous *PD*, so that the firm's asset value parameters need not be estimated. In this respect, the model combines features of both structural-form and reduced-form models. Assuming a positive correlation between a firm's asset value and collateral value, the authors obtain a similar result as Frye (2000a, 2000b), that realized default rates and recovery rates have an inverse relationship.

The model proposed by Frye (2000) draws from the conditional approach suggested by Gordy (2000), who suggests that defaults are driven by a single systematic factor – the state of the economy - rather than by a multitude of correlation parameters. Gordy's (2000) model is based on the assumption that the same economic conditions that cause defaults to rise might cause *RR*s to decline, i.e. that the distribution of recovery is different in high-default periods from low-default ones. In Frye's (2000a) model, both *PD* and *RR* depend on the state of the systematic factor. The correlation between these two variables therefore derives from their mutual dependence on the systematic factor. The intuition behind this theoretical model is relatively simple: if a borrower defaults on a loan, a bank's recovery may depend on the value of the loan collateral. The value of the collateral, like the value of other assets, depends on economic conditions. If the economy experiences a recession, *RR*s may decrease just as default rates tend to increase. This gives rise to a negative correlation between default rates and *RR*s.

While the model originally developed by Frye (2000a) implied recovery to be taken from an equation that determines collateral, Frye (2000b) modeled recovery directly. This allowed him to empirically test his model using data on defaults and recoveries from Moody's Default Risk Service database covering US obligors for the 1982-1997. Results show a strong negative correlation between default rates and *RR*s for corporate bonds. This evidence is consistent with U.S. bond market data, indicating a simultaneous increase in default rates and *LGD*s for the 1999-2002 period. Frye's (2000b and 2000c) empirical analysis allows him to conclude that in a severe economic downturn**,** bond recoveries might decline 20-25 percentage points from their normal-year average. Loan recoveries may decline by a similar amount, but from a higher level.

Using four different datasets ranging from 1970 to 1999, Carey and Gordy (2003) analyze *LGD* measures and their correlation with default rates. Their preliminary results contrast with the findings of Frye (2000b): estimates of simple default rate-*LGD* correlation are close to zero. They find**,** however**,** that limiting the sample period to 1988-1998, estimated correlations are more in line with Frye's results (0.45 for senior debt and 0.8 for subordinated debt). The authors postulate that during this short period the correlation rises not so much because *LGD*s are low during the low-default years 1993-1996, but rather because *LGD*s are relatively high during the high-default years 1990 and 1991. They therefore conclude that the basic intuition behind Frye's model may not adequately characterize the relationship between default rates and *LGD*s. Indeed, a weak or asymmetric relationship suggests that default rates and *LGD*s may be influenced by different components of the economic cycle.

Jarrow (2001) presents a new methodology for estimating *RR*s and *PD*s implicit in both debt and equity prices. As in Frye, *RR*s and *PD*s are correlated and depend on the state of the macroeconomy. However, Jarrow's methodology explicitly incorporates equity prices in the estimation procedure, allowing the separate identification of *RR*s and *PD*s and the use of an expanded and relevant dataset. In addition to that, the methodology explicitly incorporates a liquidity premium in the estimation procedure, which is considered essential in light of the high variability in the yield spreads between risky debt and U.S. Treasury securities.

Hu and Perraudin (2002) also examine the dependence between recovery rates and default rates using Moody's historical bond market data. They first standardize the quarterly recovery data in order to filter out the volatility of recovery rates due to changes over time in the pool of rated borrowers. They find that correlations between quarterly recovery rates and default rates for bonds issued by US-domiciled obligors are 0.22 for post-1982 data (1983-2000) and 0.19 for the 1971-2000 periods. Using extreme value theory and other non-parametric techniques, they also examine the impact of this negative correlation on credit VaR measures and find that the increase is statistically significant when confidence levels exceed 99%.

Using defaulted bonds' data for the sample period 1982-2002, Altman, Brady, Resti and Sironi (2005) find empirical results that appear consistent with Frye's intuition: a negative correlation between default rates and *RR*s. However, they find that the single systematic risk factor – i.e. the performance of the economy - is less predictive than Frye's model would suggest. Their econometric univariate and multivariate models assign a key role to the supply of defaulted bonds (the default rate) and show that this variable, together with variables that proxy the size of the high-yield bond market and the economic cycle, explain a substantial proportion (close to 90%) of the variance in bond recovery rates aggregated across all seniority and collateral levels. They conclude that a simple market mechanism based on supply and demand for the defaulted securities drives aggregate recovery rates more than a macroeconomic model based on the common dependence of default and recovery on the state of the cycle. In high default years, the supply of defaulted securities tends to exceed demand, thereby driving secondary market prices down. This in turn negatively affects *RR* estimates, as these are generally measured using bond prices shortly after default. During periods of low defaults, as we have observed in the 2004-2006 cycle, recoveries increase.

These findings are further supported by empiric analysis provided in Altman (2006), who further argues that there was a type of "credit bubble" causing seemingly highly distressed firms to remain non-bankrupt when, in more "normal" periods, many of these firms would have defaulted. This, in turn, produced an abnormally low default rate and the huge liquidity of distressed debt investors bidding up the prices of both existing and newly defaulted issues.

The studies described above had a nearly immediate practical impact, with the BIS (2005) suggesting guidance on separate downturn *LGD* modeling for banks. Despite little consensus currently observed in regards with this issue, it should be noted that the BIS suggests the use of ultimate recoveries and not recoveries at the time of default. As such, the correlation between default and recovery rates observed in the bond markets by several researchers, discussed earlier, may not imply a negative correlation between default and ultimate recovery rates. Indeed, there is timing disconnect which may be important, especially if the distressed loan market is not efficient and the discounted values of ultimate recoveries are materially different from the recovery values at the time of default. This once is especially the case for the banking industry, where regulators are known to support institutions with negative net assets, recoveries can turn out extremely lengthy and the market trading of defaulted liabilities may be thin or non-existent.

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